ENGINEERING 5

Lecture 5:
Forward and Inverse Kinematics; Learning Styles

Professor Carr Everbach

Course web page:
http://www.swarthmore.edu/NatSci/ceverba1/Class/e5/E5Index.html

Remember…

- Friday 10/1: 2nd MatLab lab is due (postponed due to Wednesday night Physics 3 exam)
- Thursday 10/7: 3rd MatLab lab is due (must have a robot assembled by then!)
- Monday & Wednesday & Thursday (this week): Wizards available (Trotter 201) from 7:00-10:00 p.m. specifically for E5.

An employment opportunity

The Sustainability Committee needs someone to attend meetings, take notes, post them on the SusCom website, and interact with the "Green Groups." Est. time: <3 hours/week.

- The Environmental Sustainability Programs Liaison works with the Swarthmore College Committee on Environmental Sustainability (SusCom), student groups interested in environmental issues, and the Lang Center Student Advisory Committee. Preference will be given to a student with a demonstrable interest in environmental sustainability and with web skills.

A volunteer opportunity

Professor Macken works with kids from Chester on Engineering projects (model bridges in the fall, solar cars in the spring).

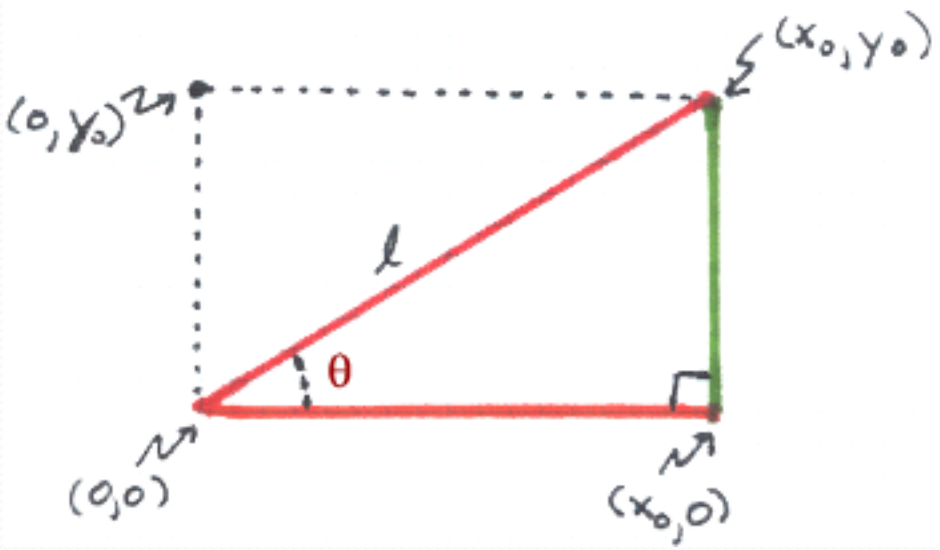
You must have Wednesday afternoons after 3:45 free.

The bridge project begins the Wednesday after break (October 20) and goes 4-5 weeks.

Contact Professor Macken, nmacken1, x8073

A volunteer opportunity

Trig review – basic functions



$$x_0 = \ell \cos(\theta) \qquad \cos(\theta) = \frac{x_0}{\ell}$$

$$y_0 = \ell \sin(\theta) \qquad \sin(\theta) = \frac{y_0}{\ell}$$

$$\tan(\theta) = \frac{y_0}{x_0}$$

Inverse functions

$$\theta = \operatorname{acos}\left(\frac{x_0}{\ell}\right)$$

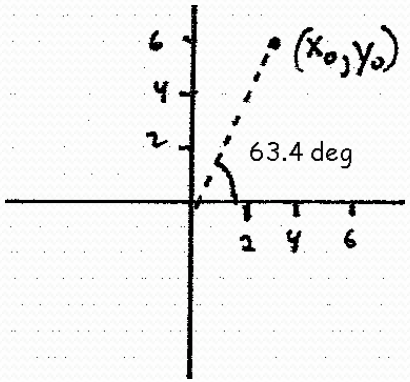
$$\theta = \operatorname{asin}\left(\frac{y_0}{\ell}\right)$$

$$\theta = \operatorname{atan}\left(\frac{y_0}{x_0}\right)$$

But, be careful with arctangent…

Trig review – arctangents

Consider a point in the first quadrant, (x_0,y_0).

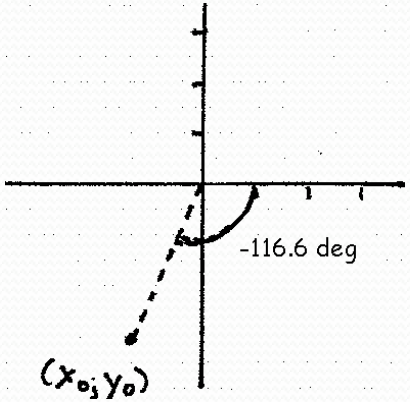


$$\tan(\theta) = \frac{y_0}{x_0} = \text{slope of line}$$

```
>> atan(6/3) * 180/pi
ans = 63.4349

>> atan2(6,3)*180/pi
ans = 63.4349
```

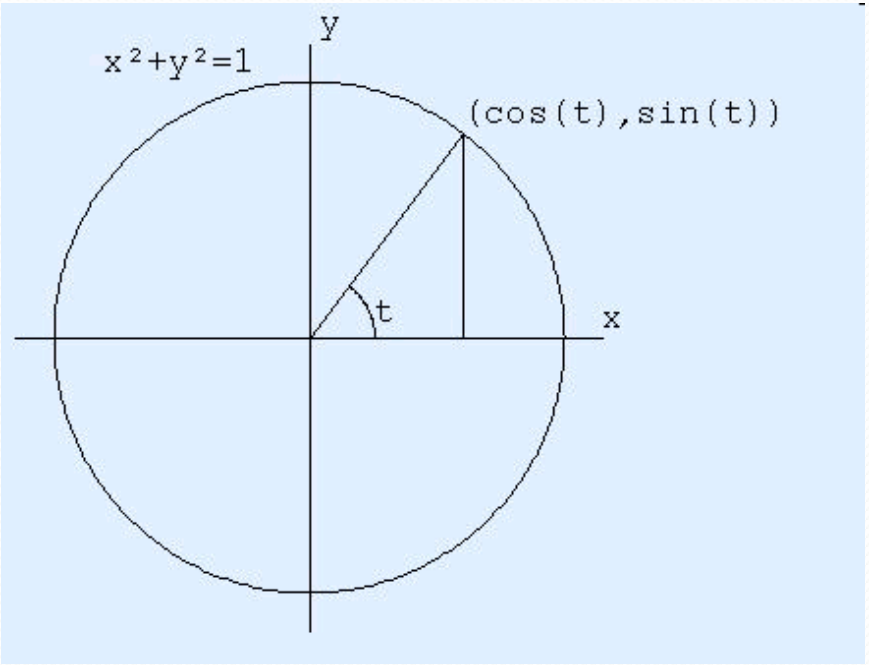
Now consider a point in the third quadrant, (x_0,y_0).




```
>> atan(-6/-3) * 180/pi
ans =   63.4349

>> atan2(-6,-3) * 180/pi
ans = -116.5651
```

Trig review – Pythagoras



www.gbbservices.com/images/cos2sin201.jpg

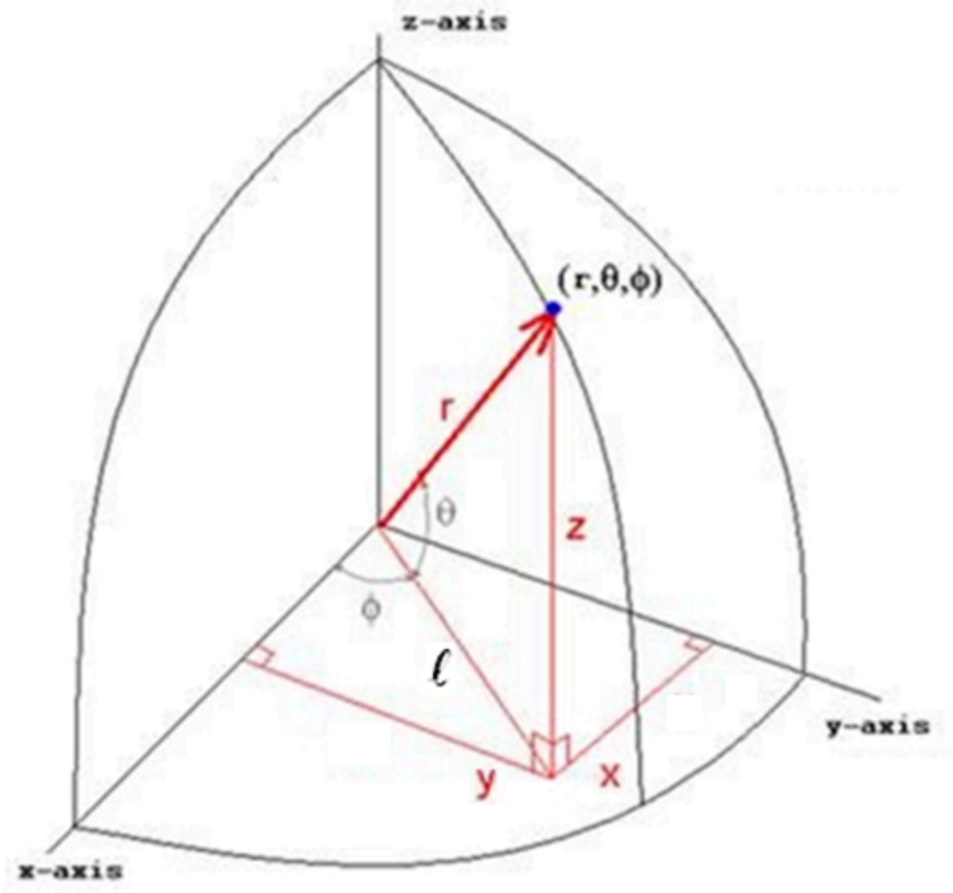
$x_0^2 + y_0^2 = \ell^2$, Pythagoras' therom

$$x_0 = l\cos\theta \qquad y_0 = l\sin\theta$$

$$l^2\cos^2\theta + l^2\sin^2\theta = l^2$$

$$\cos^2\theta + \sin^2\theta = 1$$

Spherical ⟶ Cartesian Coordinates



Adapted from: http://rbrundritt.spaces.live.com/blog/cns!E7DBA9A4BFD458C5!280.entry

$$z = r\sin(\theta)$$

$$\ell = r\cos(\theta)$$

$$x = \ell\cos(\phi) = r\cos(\theta)\cos(\phi)$$

$$y = \ell\sin(\phi) = r\cos(\theta)\sin(\phi)$$

Given r, θ and φ we can easily find x, y and z. (Forward kinematics)

Given x, y, z we could find θ and φ with some difficulty. (Inverse kinematics)

… but this is not the problem we want to solve.

2-D Polar ⟶ Cartesian w/ offset

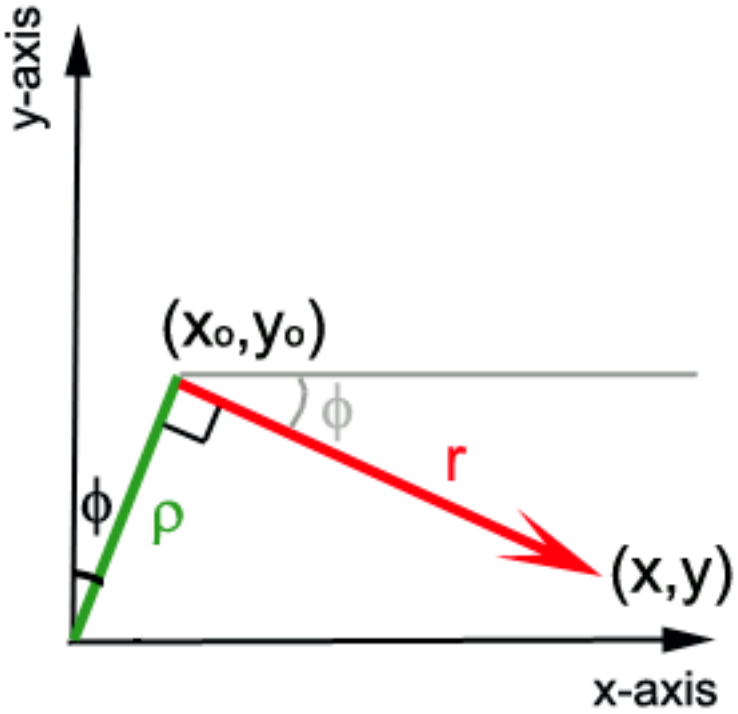
$$x_0 = \rho \sin(\phi)$$

$$y_0 = \rho \cos(\phi)$$

Angle of "r" from horizontal is equal to φ.

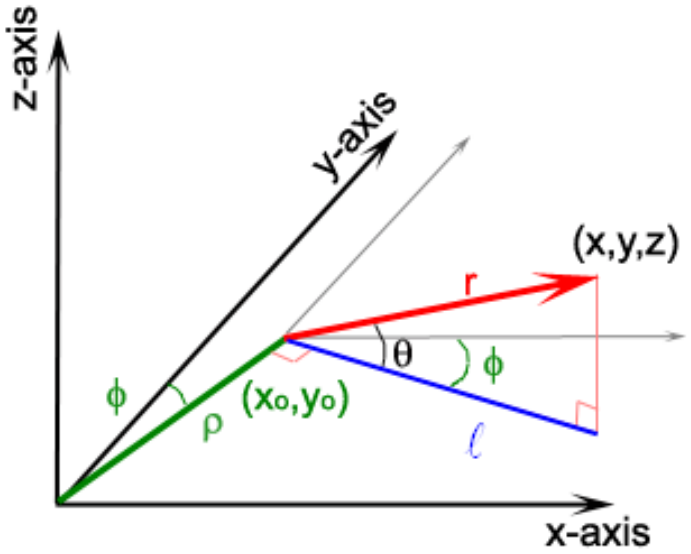
$$x = x_0 + r\cos\phi = \rho\sin\phi + r\cos\phi$$

$$y = y_0 + r\sin\phi = \rho\cos\phi + r\sin\phi$$



Forward Kinematics (1)

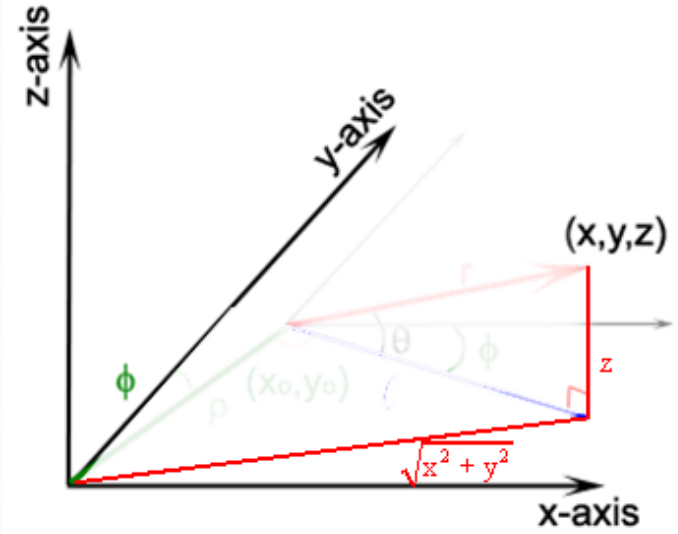
We know θ, φ and ρ.
We want to know x, y, z



Pythagoras tells us: $x^2 + y^2 + z^2 = \rho^2 + r^2$

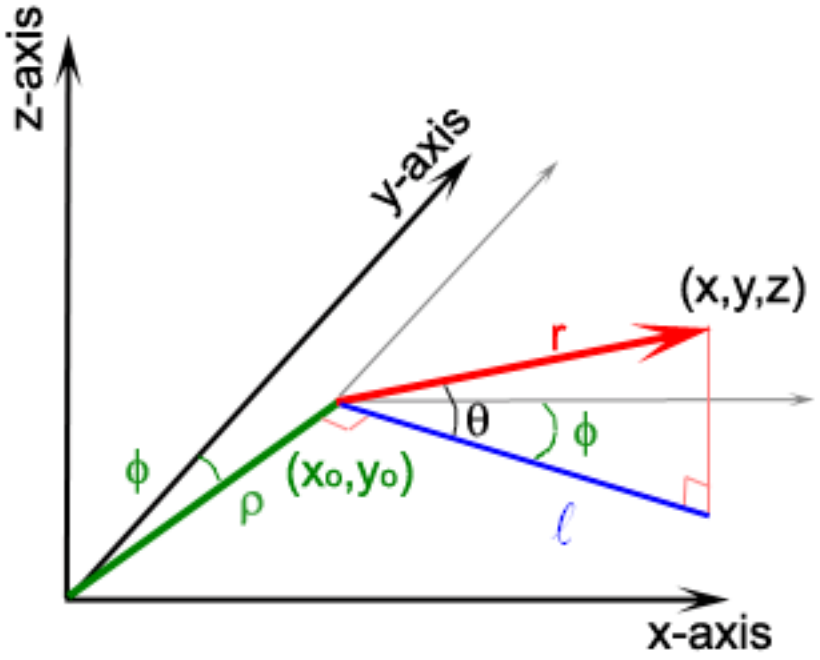
$$r = \sqrt{x^2 + y^2 + z^2 - \rho^2}$$

$$z = r\sin(\theta)$$



Forward Kinematics (2)

We know θ, φ and ρ.
We want to know x, y, z



$$x_0 = \rho \sin(\phi)$$

$$y_0 = \rho \cos(\phi)$$

$$x = x_0 + l\cos\phi$$

$$x = \rho\sin\phi + l\cos\phi$$

$$x = \rho\sin\phi + r\cos\theta\cos\phi$$

$$y = y_0 - l\sin\phi$$

Note sign in this expression for y

$$y = \rho\cos(\phi) - \ell\sin(\phi)$$

$$y = \rho\cos(\phi) - r\cos(\theta)\sin(\phi)$$

Inverse Kinematics (1)

So… using forward kinematics we can determine x, y and z, given the angles φ and θ.

$$x = \rho \sin(\phi) + r \cos(\theta)\cos(\phi) \qquad y = \rho \cos(\phi) - r \cos(\theta)\sin(\phi)$$

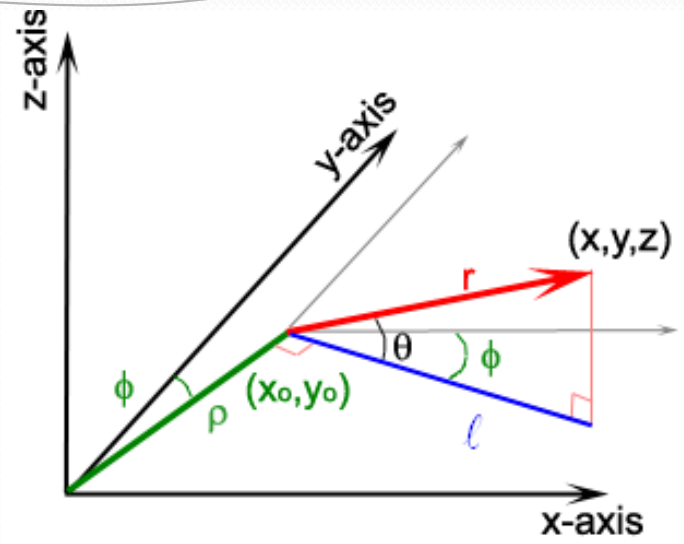
But… forward kinematics is not enough. Generally with a robot, we know where we want the robot to be (x,y), and need to find the angles.

This process is called inverse kinematics.

Problem statement: we know x, y, and z (these are inputs) and we know ρ (determined by geometry of robot).

We want to find φ and θ.

Inverse Kinematics (2)



We know x, y, z and ρ.

Solving for θ. (this is relatively easy)

We know: $z = r\sin(\theta)$

So: $\theta = \arcsin\left(\frac{z}{r}\right)$

…and we have solved for one of our two angles.

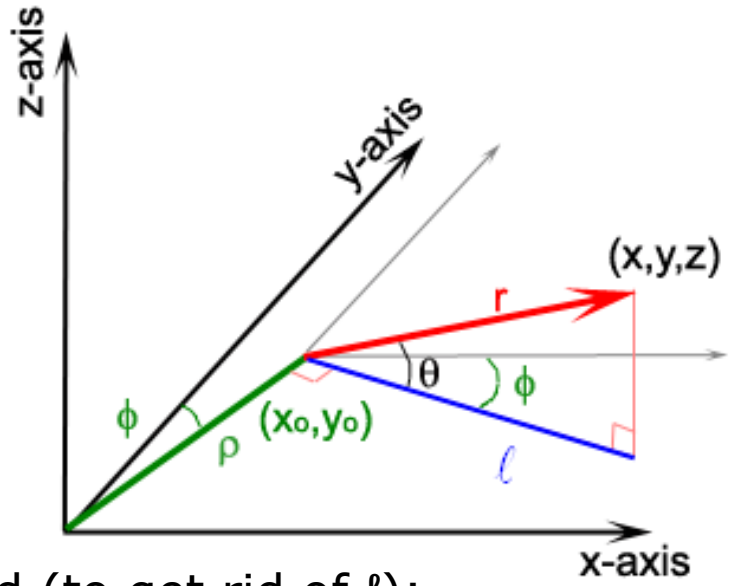
Inverse Kinematics (3)

We know x, y, z and ρ.

Solving for φ. (this is harder)

Start with: $x = \rho\sin(\phi) + \ell\cos(\phi)$

$y = \rho\cos(\phi) - \ell\sin(\phi)$



Multiply x by sin(φ) and y by cos(φ) and add (to get rid of ℓ):

$$x\sin(\phi) = \rho\sin^2(\phi) + \ell\cos(\phi)\sin(\phi)$$

$$y\cos(\phi) = \rho\cos^2(\phi) - \ell\sin(\phi)\cos(\phi)$$

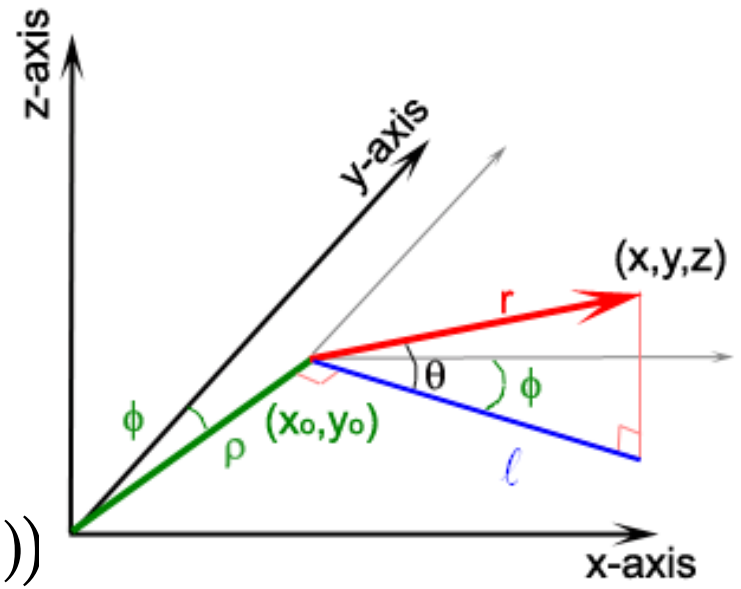
$$x\sin(\phi) + y\cos(\phi) = \rho\sin^2(\phi) + \cancel{\ell\cos(\phi)\sin(\phi)} + \rho\cos^2(\phi) - \cancel{\ell\sin(\phi)\cos(\phi)}$$

$$x\sin(\phi) + y\cos(\phi) = \rho\left(\sin^2(\phi) + \cos^2(\phi)\right)$$

$$x\sin(\phi) + y\cos(\phi) = \rho$$

We're almost done…

Inverse Kinematics (4)



… continued from previous

$$x\sin(\phi) + y\cos(\phi) = \rho$$

Use trigonometric identity:

$$a\sin(\alpha) + b\cos(\alpha) = \sqrt{a^2 + b^2}\,\sin(\alpha + a\tan2(b,a))$$

$a = x \qquad b = y \qquad \alpha = \phi$

$$x\sin(\phi) + y\cos(\phi) = \sqrt{x^2 + y^2}\,\sin(\phi + a\tan2(y,x)) = \rho$$

$$\sin(\phi + a\tan2(y,x)) = \frac{\rho}{\sqrt{x^2 + y^2}}$$

$$\phi + a\tan2(y,x) = a\sin\left(\frac{\rho}{\sqrt{x^2 + y^2}}\right)$$

so…

$$\phi = a\sin\left(\frac{\rho}{\sqrt{x^2 + y^2}}\right) - a\tan2(y,x)$$

Servo Motors (1)

- Output shaft of motor turns to angle specified by input pulses.
- We send a stream of pulses to servo through wires connected to it.
- Width of pulses defines angle of output shaft.

http://www.lynxmotion.com/images/Products/Full/hsr-5980.jpg

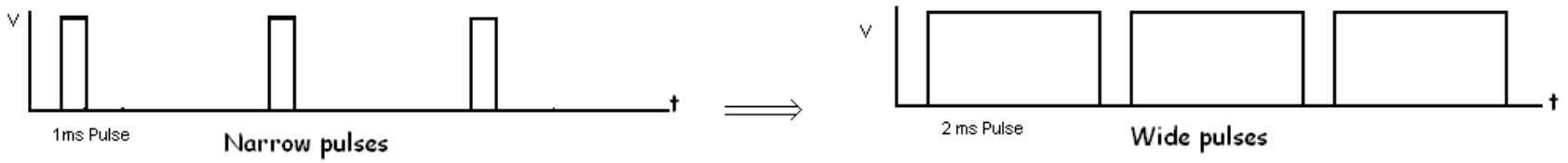
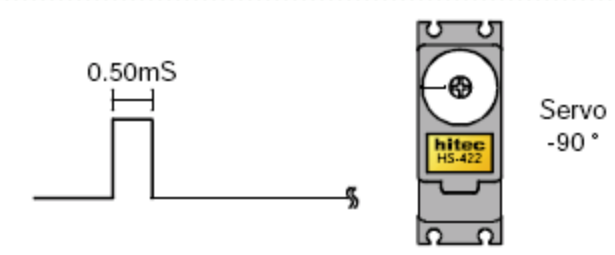
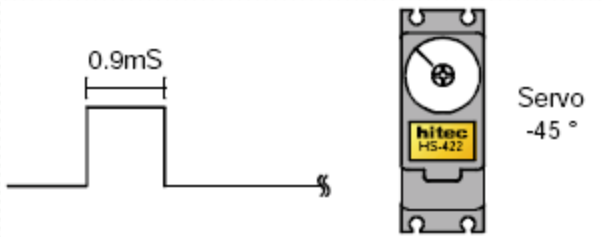


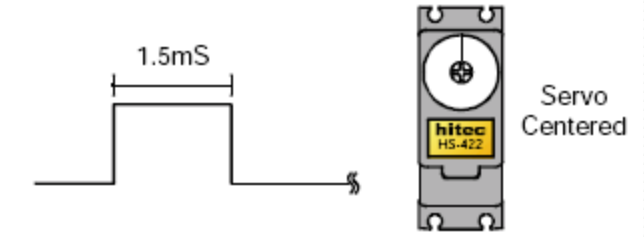
Image adapted from: http://www.electronicsteacher.com/robotics/images/pulsewidthdiagram.gif

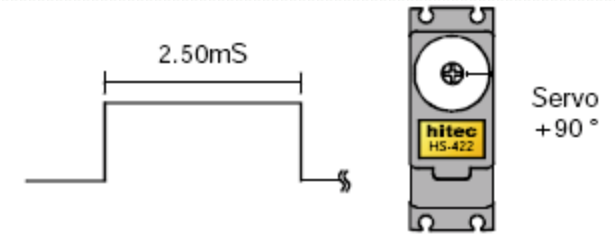
Servo Motors (2)

Relatively precise, but not accurate









Images adapted from: http://www.lynxmotion.com/images/data/ssc-32.pdf

Controlling the Servo

- Open a serial port connection
- Send a string of characters to the I/O board
- The I/O board controls the motors by varying the width of pulses sent to the servo

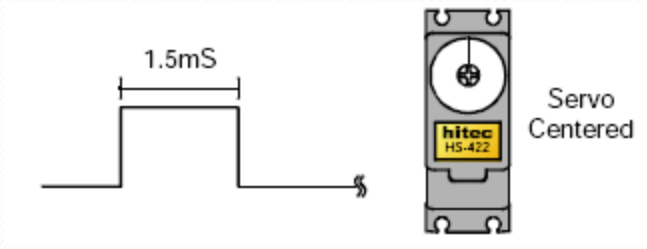
Control Format

- #<ch>P<pw>...#<ch>P<pw>T<time><cr>
- <ch> = channel number (0-31)
- <pw> = pulse width in microseconds (500-2500)
- <time> = time in milliseconds for move (optional)
- <cr> = carriage return http://www.youtube.com/watch?v=Pp2IJIk5qvc

Control Format Examples

#<ch>P<pw>...#<ch>P<pw>T<time><cr>

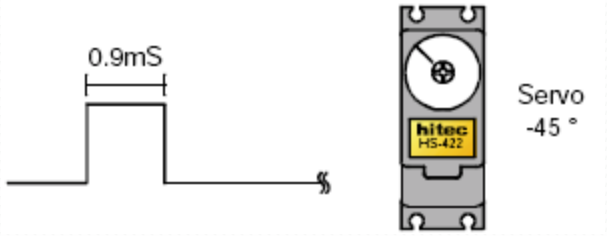
- Sending the string: '#1P1500T200<cr>'
  - ch=1, pw=1500, time=200
  - This will set the pulsewidth on motor 1 to 1500 microseconds, and will make the change over 200 milliseconds. (1500 ms ≈ 0°)



Another Control Format Examples

#<ch>P<pw>...#<ch>P<pw>T<time><cr>

- Sending the string: '#1P900T300<cr>'
  - ch=1, pw=900, time=300
  - This will set the pulsewidth on motor 1 to 900 microseconds, and will make the change over 300 milliseconds. (900 ms ≈ -45°)



Control Format Example Again

#<ch>P<pw>...#<ch>P<pw>T<time><cr>

- Sending the string:
  '#1P1200#5P2100T1500<cr>'
  - ch=1, pw=1200
    ch=5, pw=2100
    time=1500
  - This will set
    - the pulsewidth on motor 1 to 900 microseconds,
    - the pulsewidth on motor 5 to 2100 microseconds,
    - and will make the change over 1500 milliseconds.

Some MatLab Background

Open a serial connection (done only once).

```
s=instrfind;     %Find any serial links (we can have only 1)
delete(s);       %... and delete.

%Create a new serial communications link
s=serial('COM1','Baudrate',115200,'Terminator','CR');
fopen(s);                                %... and open it
```

Send a string to I/O board.

```
% set pulsewidth on motor 1 to 1000 microseconds (over 200 mS)
fprintf(s,'#1P1000T200');
```

Learning Styles

Learning Styles

- Everybody learns in a different way.
- Learning style index, similar to Myers-Briggs personality type index (Extrovert-Introvert, Sensing-Intuition, Thinking-Feeling, Judging-Perceiving).
- Learning styles (Active-Reflective, Sensing-Intuitive, Visual-Verbal, Sequential-Global).
- Exact "axes" aren't important, but being aware of the way you learn can be very important.

Information on learning styles taken from: http://www4.ncsu.edu/unity/lockers/users/f/felder/public/ILSdir/styles.htm

Active vs. Reflective (Description)

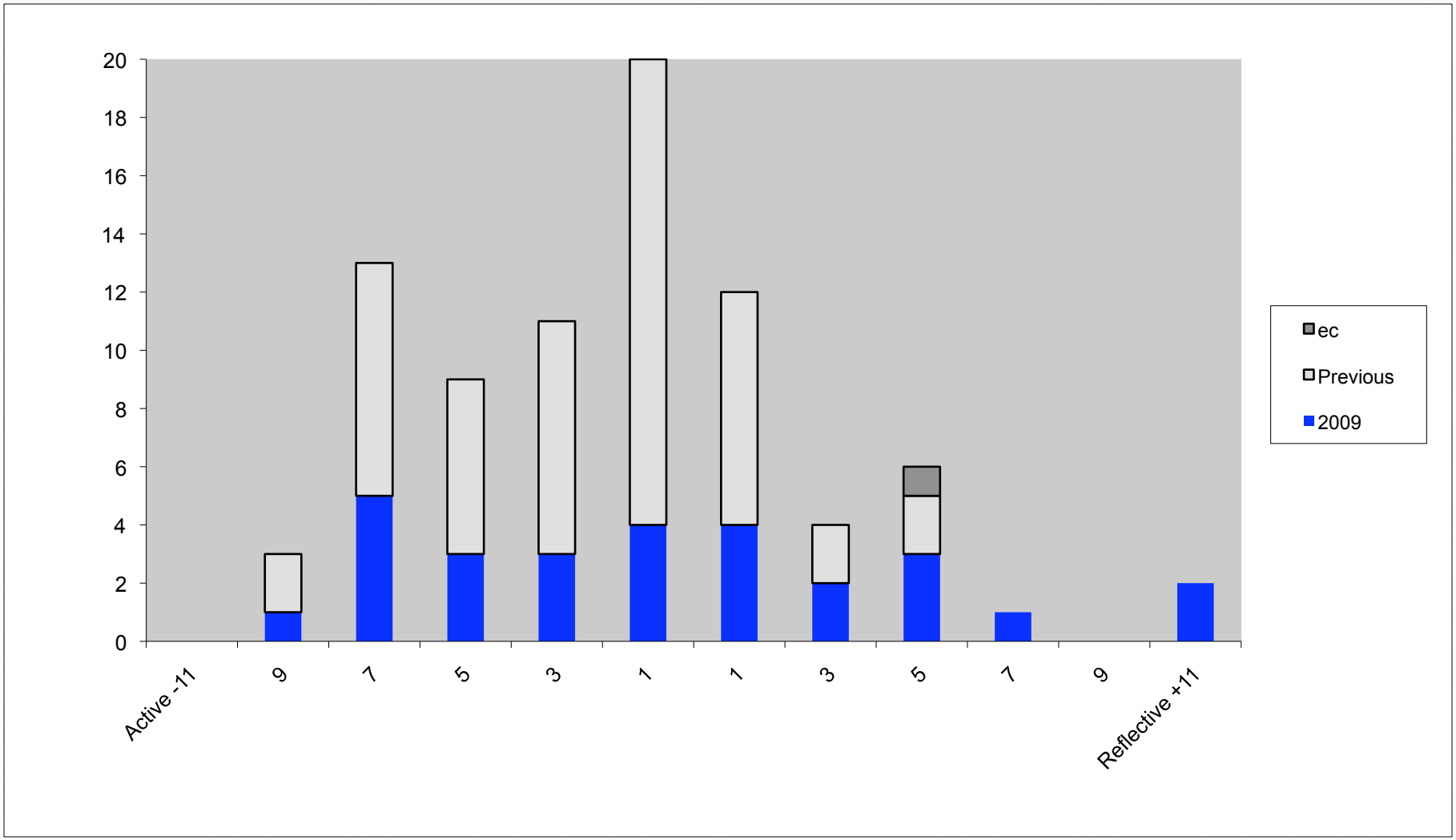
Active	Reflective
Retain information best by doing something active.	Prefer to think about it quietly first.
"Let's try it out and see how it works"	"Let's think it through first"
Group work	Working alone

Active vs. Reflective (Strategies)

Active	Reflective
Study in a group.	Stop periodically to review what you have read and think of possible questions or applications.
Quiz each other	Write short summaries of readings or class notes in your own words.

Active vs. Reflective

Average = -1.6



Sensing vs. Intuitive (Description)

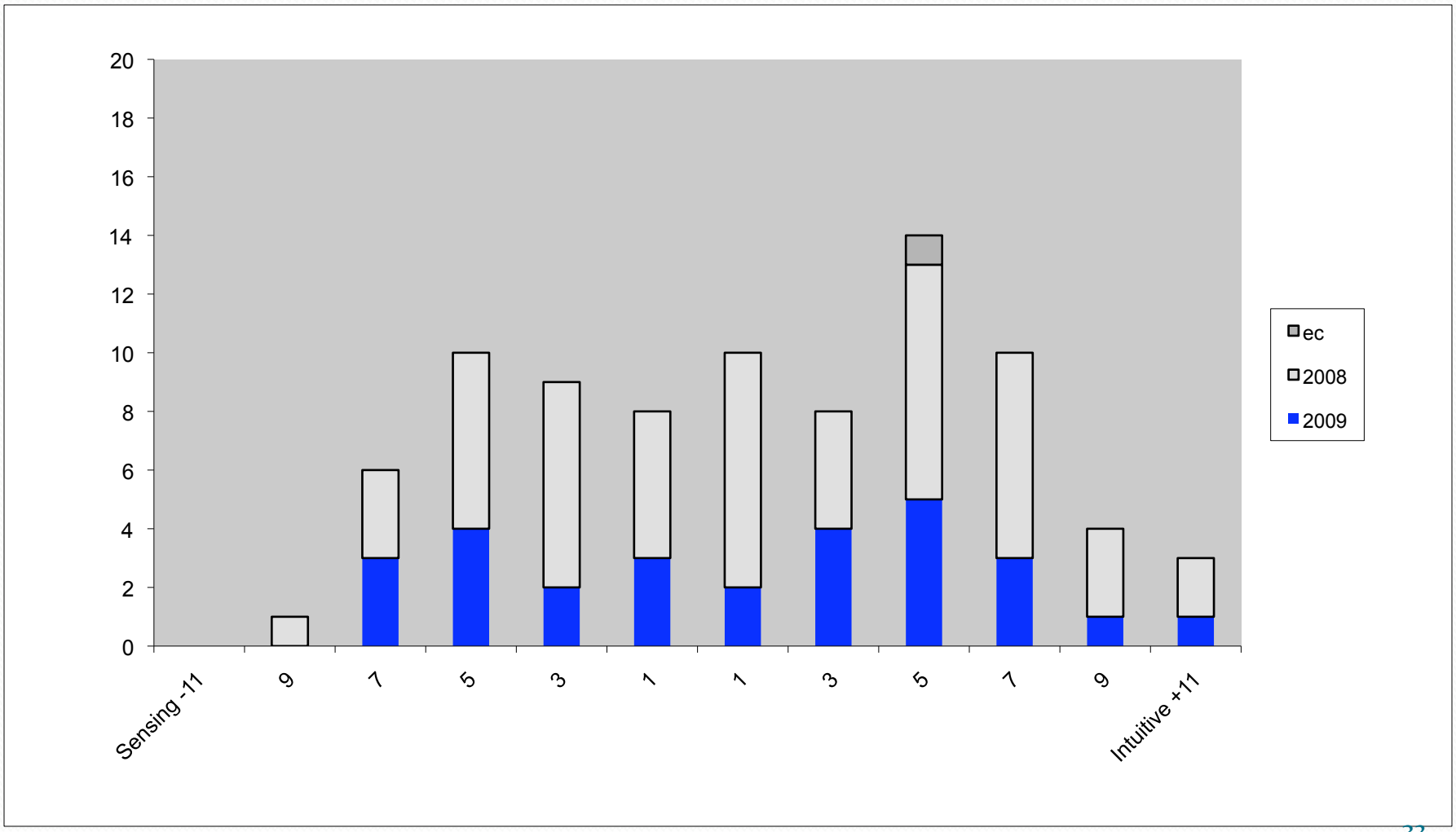
Sensing	Intuitive
like learning facts	prefer discovering relationships
solve problems by well-established methods	like innovation and dislike repetition
practical and careful	work quickly and innovatively
like connections to real world	don't like "plug-and-chug" courses w/ memorization
patient with details and good at memorizing facts	good at grasping new concepts and abstractions

Sensing vs. Intuitive (Strategies)

Sensing	Intuitive
ask instructor for specific examples of concepts and procedures	ask your instructor for interpretations or theories that link the facts
try to find specifics in textbook or other references or by brainstorming with friends or classmates	take time to read entire question before you start answering and be sure to check your results

Sensing vs. Intuitive

Average = 1.3



Visual vs Verbal (Description)

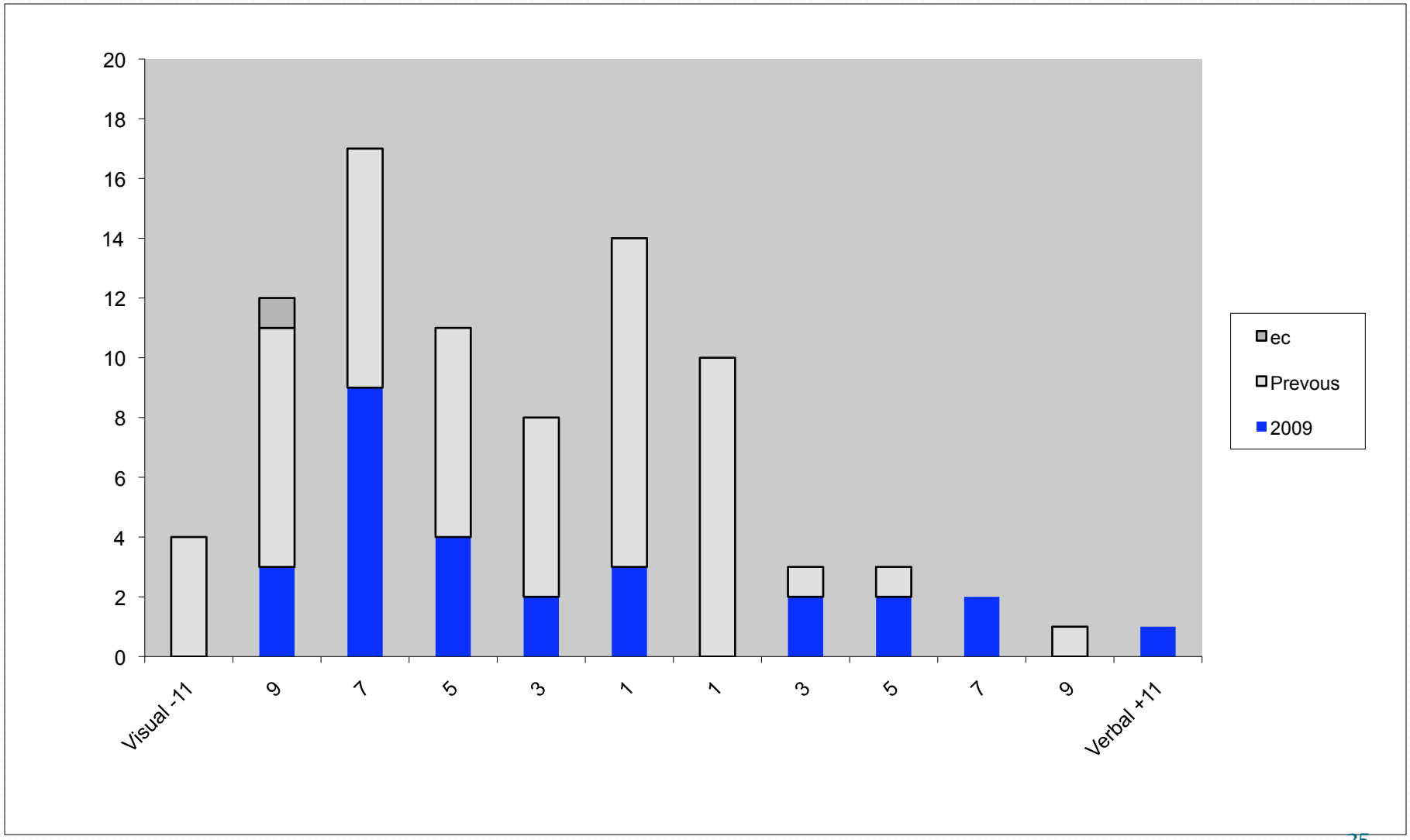
Visual	Verbal
remember best what they see--pictures, diagrams, flow charts, time lines, films, and demonstrations.	get more out of words--written and spoken explanations.

Visual vs Verbal (Strategies)

Visual	Verbal
try to find diagrams, sketches, schematics, photographs,…	write summaries or outlines of course material in your own words
see if any of the course material is available.	working in groups can be particularly effective: hearing classmates' explanations and even more when you explain
make concept map by listing key points, enclosing them in boxes or circles, and connecting	
Color-code your notes with highlighter	

Visual vs. Verbal

Average = -3.5



Sequential vs Global (Description)

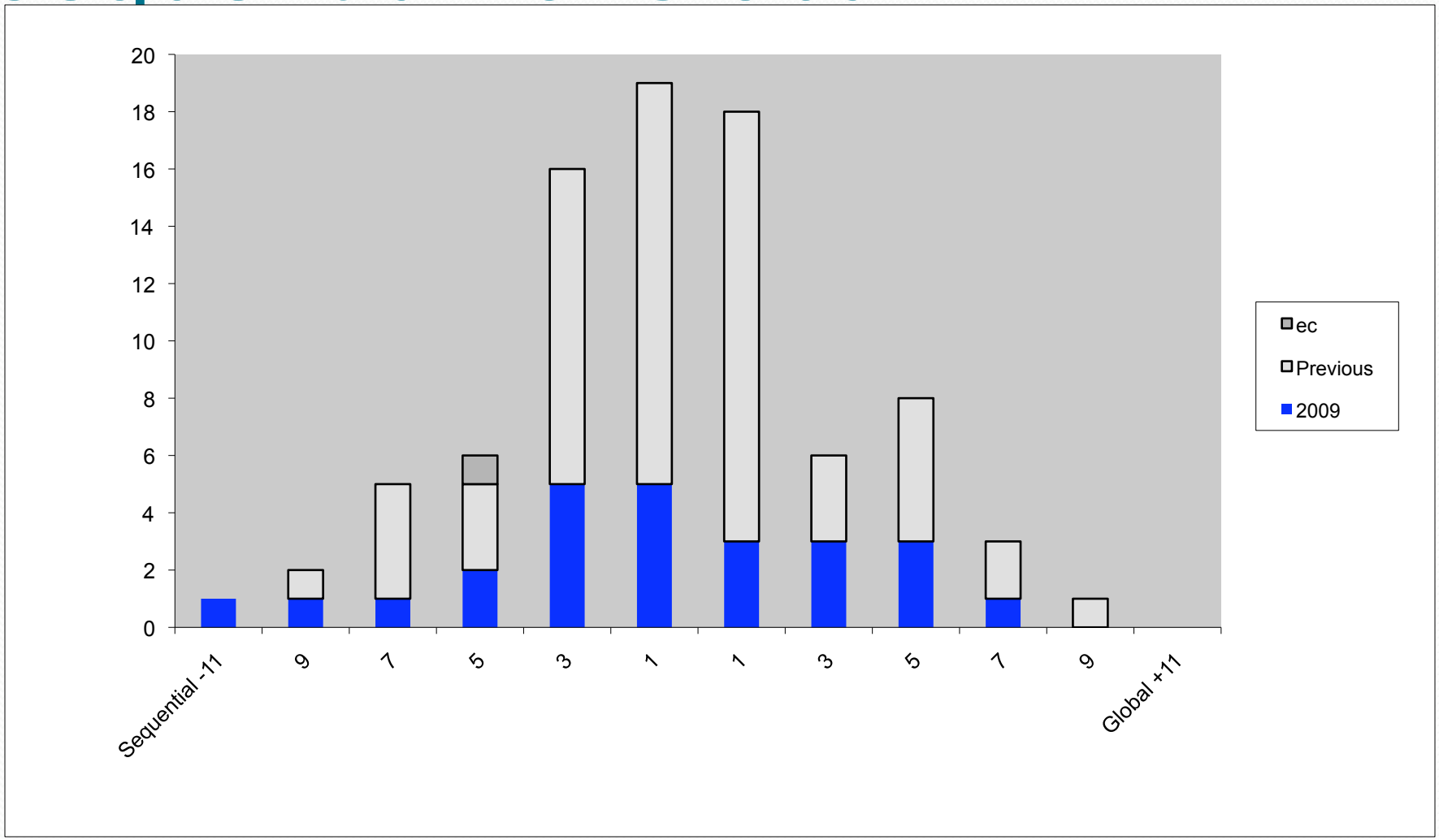
Sequential	Global
gain understanding in linear steps; each step following logically from the previous one	learn in large jumps, absorbing material almost randomly without connections, and then suddenly "getting it"
follow logical stepwise paths in finding solutions	may solve complex problems quickly or in novel ways once they have big picture, but may have difficulty explaining how they did it

Sequential vs Global (Strategies)

Sequential	Global
ask the instructor to fill in the skipped steps do so yourself	skim entire chapter to get overview
outline lecture material for yourself in logical order	immerse yourself in individual subjects for large blocks instead of short bursts
	relate the subject to things you already know

Sequential vs. Global

Average = -0.6



The 5th axis: Inductive-Deductive

- '…the "best" method of teaching … is induction, whether it be called problem-based learning, discovery learning, inquiry learning, or some variation on those themes.'
- 'On the other hand, the traditional college teaching method is deduction, starting with "fundamentals" and proceeding to applications.
- 'I don't want instructors to be able to determine somehow that their students prefer deductive presentation and use that result to justify continuing to use the traditional but less effective lecture paradigm in their courses and curricula. I have therefore omitted this dimension from the model.'