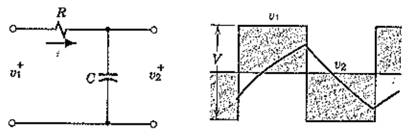


(a) Circuit (b) Waveforms

Figure 3.38 Integrating circuit.

If the time constant RC is large compared to the period T of the square waves, only the straight portion of the exponential appears and the output is the *sawtooth* wave in which voltage is directly proportional to time. In general,

$$v_1 = v_R + v_C \cong v_R = iR \tag{3-36}$$

if v_C is small compared to v_R (i.e., $RC > T$). Then

$$v_2 = \frac{1}{C}\int i\,dt \cong \frac{1}{RC}\int v_1\,dt \tag{3-37}$$

and the output is approximately proportional to the integral of the input. If it is necessary, the magnitude of the signal can be restored by linear amplification.

OP AMP INTEGRATOR AND DIFFERENTIATOR

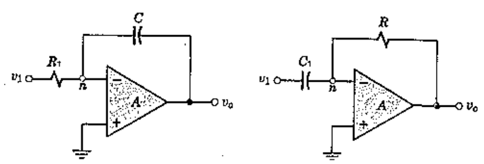
In the previous discussion of op amps we assumed that the feedback network is purely resistive. In general, however, the network may contain capacitances, inductances, and resistances. Because of the high amplifier gain in combination with feedback, the mathematical operations performed are precise.

In Fig. 3.39a, the feedback element is a capacitor. For the op amp, $v_i \cong 0$, node n is at ground potential, $i_i \cong 0$, and the sum of the currents into node n is

$$\frac{v_1}{R_1} + C\frac{dv_o}{dt} = 0 \quad \text{or} \quad dv_o = -\frac{v_1}{R_1 C}\,dt \tag{3-38}$$

Integrating each term with respect to time and solving,

$$v_o = -\frac{1}{R_1 C}\int v_1\,dt + \text{a constant} \tag{3-39}$$



(a) Integrator (b) Differentiator

Figure 3.39 Op amp integration and differentiation circuits.

and the device is an *integrator*. The analog integrator is very useful in computing, signal processing, and signal generating.

Example 13

Predict the output voltage of the circuit shown in Fig. 3.40 where the block represents an ideal amplifier with A very large.

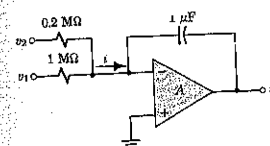


Figure 3.40 Integrator application.

For A very large, v_i is very small and representation as an integrator is accurate. Here $i = i_1 + i_2$ and

$$v_o = -\frac{1}{C}\int\left(\frac{v_1}{R_1} + \frac{v_2}{R_2}\right)dt$$

$$= -\frac{1}{CR_1}\int\left(v_1 + v_2\frac{R_1}{R_2}\right)dt$$

$$= -\int(v_1 + 5v_2)\,dt$$

The output is the integral of a weighted sum.

If the resistance and capacitance are interchanged as in Fig. 3.39b, the sum of the currents is

$$C_1\frac{dv_1}{dt} + \frac{v_o}{R} = 0 \tag{3-40}$$

Solving,

$$v_o = -RC_1\frac{dv_1}{dt} \tag{3-41}$$

and the output voltage is proportional to the derivative of the input. For practical reasons involving instability and susceptibility to noise, the differentiator is not so useful as the integrator.

Exercise 3-14

For Example 13, let $v_1 = e^{-2t}$ and $v_2 = e^{-100t}$ for $t \geq 0$ and $v_o = 0$ at $t = 0$. Find and sketch v_o over the period $0 < t < 2.5$ s.

Answer: $v_o = 0.5\,e^{-2t} + 0.05\,e^{-100t} - 0.55,\ t \geq 0.$

SUMMARY

- Exponentials and sinusoids are important waveforms because: they occur frequently, they are easy to handle mathematically, and they are used to represent other waves.
  The general decaying exponential is $a = A\,e^{-t/\tau}$
  The time constant τ is a measure of the rate of decay.
  For $t = \tau$, $a/A = 1/e = 0.368$; for $t = 5\tau$, $a/A = 0.0067$ (negligible).

- The general sinusoid is $a = A \cos(\omega t + \alpha)$.
  Frequency $\omega = 2\pi f$ rad/s; $f = 1/T$ Hz, where period T is in seconds.
- In a periodic function of time, $f(t + T) = f(t)$.
  The average value of a periodic current is $I_{av} = (1/T) \int_0^T i\, dt$.
  For a sinusoid, the half-cycle average is $2I_m/\pi = 0.637I_m$.
  The effective or rms value of a periodic current is $I = \sqrt{(1/T) \int_0^T i^2\, dt}$.
  For a sinusoid, the effective value is $I = I_m/\sqrt{2} = 0.707I_m$.
- An ideal amplifier is characterized by: infinite input resistance, zero output resistance, and constant gain; an ideal op amp has infinite gain.
  Feedback circuits are used with op amps to obtain: inverting and noninverting amplifiers, summing circuits, integrators, and differentiators.
- Essentially, a diode discriminates between forward and reverse voltages.
  The ideal diode presents zero resistance in the forward direction and infinite resistance in the reverse direction; it functions as a selective switch.
- A rectifier converts alternating current into unidirectional current.
  In half-wave rectification with a resistive load, $I_{dc} \cong V_m/\pi R_L = I_m/\pi$.
  A bridge circuit or phase inverter permits full-wave rectification; $I_{dc} = 2I_m/\pi$.
- A capacitor filter stores charge on voltage peaks and delivers charge during voltage valleys; the ripple voltage (half-wave) is $V_r \cong V_{dc}/fCR_L$.
- Waveforms can be shaped easily, rapidly, and precisely.
  A diode-resistor-battery circuit can perform clipping.
  A diode and peak-charging capacitor can clamp signals to desired levels.
  An op amp can perform differentiation and integration precisely.

TERMS AND CONCEPTS

diode Two-terminal device that acts as a switch; it permits current to flow readily in one direction but tends to prevent the flow of current in the other direction.

effective value of a current Steady current that is as effective in transferring power as the given varying current.

filter Circuit passing signals of selected frequencies while rejecting signals of different frequencies.

operational amplifier Amplifier with a high gain designed to be used with other circuit components to perform a specified signal-processing function.

rectifier Device for changing alternating current to unidirectional current.

signal, electric Voltage or current varying with time in a manner that conveys information.

sinusoidal waveform Voltage or current variation in accordance with a sine or cosine function of time.

time constant Measure of the rate of decay of an exponential function; equal to the time for an exponential response to decrease to 37% of its initial value.

waveform Pattern of time variation of a voltage or current.

REVIEW QUESTIONS

1. Write the general equation of an exponentially *increasing* function.
2. Why is the "time constant" useful?
3. Justify the statement: "In a linear circuit consisting of resistances, inductances, and capacitances, if any voltage or current is an exponential all voltages and currents will be exponentials with the same time constant."
4. Write a statement for sinusoids similar to the foregoing statement for exponentials and justify it.
5. Distinguish between average and effective values.
6. Why are ac ammeters calibrated to read rms values?
7. What is the reading of a dc ammeter carrying a current $i = 10 \cos 377t$ A?
8. What is "amplification"? "Gain"?
9. Why are *high* input resistance and *low* output resistance desirable in an amplifier?
10. What is a differential amplifier? What are its advantages?
11. What is an operational amplifier?
12. Why is the gain of an amplifier circuit independent of the gain of the op amp employed?
13. In circuit analysis of an ideal op amp with feedback, what values are assumed for v_i and i_i? Why?
14. How can an op amp be used to add two analog signals?
15. Why do we wish to replace actual devices with fictitious models?
16. Explain the operation of a full-wave bridge rectifier.
17. Explain the operation of a capacitor filter after a full-wave rectifier.
18. During capacitor discharge, why doesn't some of the current flow through the diode?
19. What is the effect on the ripple factor of the circuit in Fig. 3.30 of doubling R_L? Of halving C? Of doubling V_m?
20. Explain the operation of a diode clipper.
21. Sketch a clamping circuit and explain its operation.
22. Explain in words the operation of a differentiator and an integrator.

PROBLEMS

P3-1. Sketch the following signals and write mathematical expressions for $v(t)$:
(a) A step of 2 V occurring at $t = 1$ s.
(b) A sawtooth of amplitude 50 V that resets 20 times a second.
(c) A train of pulses of amplitude 100 V, duration 10 μs, and frequency 40 kHz.
(d) An exponential voltage with an initial amplitude of 5 V that decays to 1 V in 80 ms. Predict its value at $t = 20$ ms.

P3-2. An exponential voltage has a value of 15.5 V at $t = 1$ s and a value of 2.1 V at $t = 5$ s.
(a) Derive the equation for $v(t)$.
(b) How many time constants elapse between $t = 5$ s and $t = 9$ s?
(c) Predict the value at $t = 9$ s.

P3-3. An exponential current has a value of 8.7 mA at $t = 2$ s and a value of 3.2 mA at $t = 5$ s. Write the equation for $i(t)$ and predict the value at 11 s.

P3-4. Throwing a switch in an experimental circuit develops a voltage $v = 6\, e^{-500t}$ V across a parallel combination of $R = 40\ \Omega$ and $C = 20\ \mu$F. Predict the current in each element and the total current.

P3-5. Throwing a switch in an experimental circuit introduces a transient voltage $v = 120\, e^{-10t}$ V. If measurements in the circuit are accurate to 0.5%, approximately how long does the switching transient "last"?

P3-6. A current $i = 2\, e^{-250t}$ mA flows through a series combination of $R = 3$ kΩ and $L = 4$ H. Determine the voltage across each element and the total voltage.

P3-7. A 1-MΩ resistor is connected across a 500-μF capacitor initially charged to 100 V. Predict how long it will take for the voltage to drop to 14 V.

P3-8. Sketch, approximately to scale, the following functions:

(a) $i_1 = 2 \cos (377t + \pi/4)$ A
(b) $v_1 = 10 \cos (300t - \pi/3)$ V
(c) $i_2 = 1.5 \sin (400t - 60°)$ A

P3-9. Express the following sinusoidal signals as specific functions of time:

(a) A current with a frequency of 100 Hz and an amplitude of 10 mA and passing through zero with positive slope at $t = 1$ ms.
(b) A voltage with a period of 20 μs and passing through a positive maximum of 5 V at $t = 5$ μs.
(c) A current reaching a positive maximum of 2 A at $t = 5$ ms and the next negative maximum at $t = 25$ ms.
(d) A voltage having a positive maximum of 50 V at $t = 0$ and decreasing to a value of 25 V at $t = 2$ ms.

P3-10. A sinusoidal signal has a value of −5 A at time $t = 0$ and reaches its first maximum (negative) of −10 A at $t = 2$ ms. Write the equation for $i(t)$ and predict the value at $t = 5$ ms. Determine the frequency and period.

Answer: $f = 83.33$ Hz, period = 12 ms.

P3-11. A current $i = 2 \cos (2000t - \pi/4)$ A flows through a series combination of $R = 20\ \Omega$ and $L = 10$ mH.

(a) Determine the voltage across each element.
(b) Sketch the two voltages, $v = f(\omega t)$, approximately to scale and estimate the amplitude and phase angle of $v_R + v_L$.

P3-12. For each of the periodic voltage and current signals shown in Fig. P3.12:

(a) Calculate average values of voltage, current, and power.
(b) Calculate effective values of voltage and current.

P3-13. Calculate average and effective values of voltage and current for each of the periodic signals shown:

(a) In Fig. P3.13a.
(b) In Fig. P3.13b.

P3-14. A signal consists of a series of positive rectangular pulses of magnitude V, duration T_d, and period T. Where $D = T_d/T$, derive expressions for average and effective values in terms of D and V.

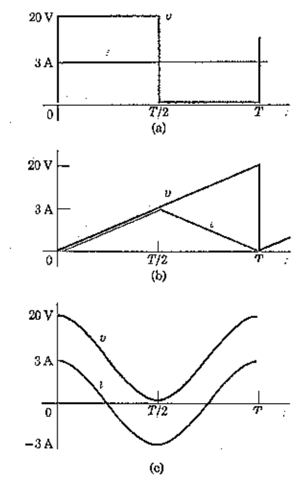


Figure P3.12 Voltage and current signals.

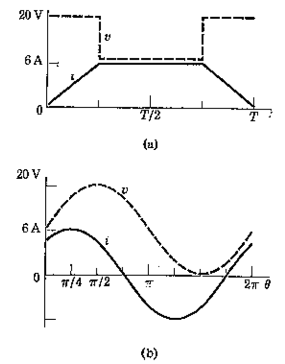


Figure P3.13 Periodic signals.

P3-15. The signal of Fig. P3.15a is applied to the circuit shown. The rectifier permits current flow only in the direction indicated by the triangle. Predict the readings on the two meters, if they are:

(a) dc ammeters
(b) ac ammeters of the types described in the text

Answer: (a) $I_1 = 0$, $I_2 = 1$ mA;
(b) $I_1 = 2.45$ mA, $I_2 = 1.73$ mA.

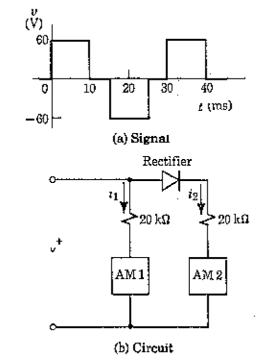


Figure P3.15

P3-16. Repeat Problem P3-15 if the voltage v is as shown in Fig. P3.16.

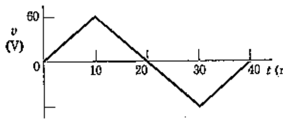


Figure P3.16

P3-17. The duty cycle of an electric motor is: At $t = 0$ (motor idling and consuming negligible power), load increases abruptly to 60 hp and holds for 10 s, load decreases uniformly to 20 hp over 10 s and then is removed; motor idles for 40 s. Sketch the duty cycle (hp versus time) and specify the proper motor (available in 10-, 20-, 30-, 50-, and 100-hp ratings).

P3-18. An op amp has a very high gain, a very high input resistance, and a very low output resistance. Design and draw an amplifier circuit (with $R_F = 3$ MΩ) providing:

(a) $A_F = -150$
(b) $A_F = -30$
(c) $A_F = +40$
(d) $A_F = +120$

P3-19. In Fig. 3.20a, $R_1 = 1$ kΩ, $R_2 = 2$ kΩ, $R_n = 3$ kΩ, and $R_F = 90$ kΩ. If $v_1 = +1$ mV, $v_2 = -5$ mV, and $v_n = +2$ mV, predict v_o.

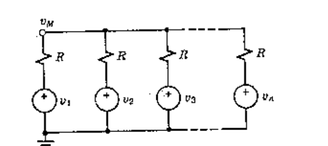


Figure P3.20

P3-20. (a) Derive an expression for voltage v_M in terms of the other voltages in Fig. P3.20. Describe in words the function performed by this circuit.
(b) Design an op amp circuit to perform the same function.

P3-21. (a) Using an ideal amplifier, design a circuit (let $R_F = 1000\ \Omega$) to provide an output $v_o(t) = -v_1(t) - 10\ v_2(t)$.
(b) Redesign the circuit to provide an output $v_o(t) = +v_1(t) - 10\ v_2(t)$.

P3-22. Given the op amp in Fig. P3.22:

(a) Define v_o in terms of the given inputs.
(b) Describe the operation performed assuming sinusoidal inputs.

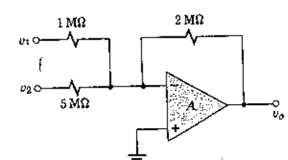


Figure P3.22

P3-23. Given the circuit of Fig. P3.23 on p. 106:

(a) Define v_o in terms of v_1.
(b) Define the operation performed.

Answers: (a) $v_o = 51v_1$;
(b) noninverting amplification.

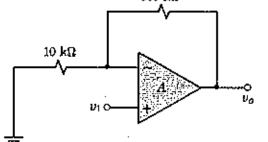


Figure P3.23

P3.24. In Fig. P3.24, $v = 10 \sin \omega t$ V and $V_B = 6$ V. Under what circumstances will current i flow? Sketch v and i as functions of time on the same axes.

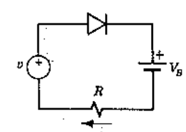


Figure P3.24

P3-25. Explaining your reasoning and stating any simplifying assumptions, predict current I in Fig. P3.25.

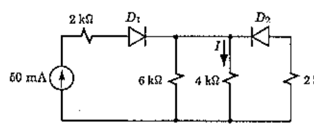


Figure P3.25

P3-26. Four ideal diodes are connected in the fullwave rectifier of Fig. 3.27 where R_L = 1000 Ω. Specify the amplitude of v so that the *average* current through R_L is 20 mA.

P3-27. An ideal diode is used in a half-wave rectifier with power supplied at 120 V (rms) and 60 Hz. For a load R_L = 2000 Ω, predict I_{dc}, V_{dc}, and the power delivered to the load.

P3-28. Repeat Problem 3-27 assuming a full-wave bridge rectifier circuit.

P3-29. A diode is connected in series with a 30-V rms source to charge a 12-V battery with an internal resistance of 0.1 Ω. Specify the series resistance necessary to limit the peak current to 2 A. Estimate the time required to recharge a 10-A·hr battery.

Answer: R = 14.66 Ω, time = 19.2 h.

P3-30. In Fig. 3.30, C = 100 μF and R_L = 10 kΩ. For V_m = 20 V at 60 Hz, predict:

(a) The dc load current through R_L.
(b) The percent ripple in v_L.
(c) The reading of a dc ammeter in series with R_L if C is disconnected.

P3-31. A "load" requires 10 mA at 30 V dc with no more than 0.5 V ripple.

(a) Draw the circuit of a power supply consisting of a transformer with 120-V 60-Hz input, half-wave rectifier, capacitor filter, and effective R_L; specify C and the turn ratio of the transformer.
(b) Repeat for a full-wave bridge rectifier circuit. Draw a conclusion.

P3-32. In Fig. 3.32b, R = 1 kΩ, V_A = 2 V, and V_B = 5 V. Sketch v_o for $v_i = 6 \sin \omega t$ V.

P3-33. In Fig. P3.33, $v_1 = 10 \sin \omega t$ V.

(a) Sketch $v_1(t)$ and $v_2(t)$ on the same graph.
(b) Draw the transfer characteristic v_o versus v_i.
(c) What function is performed by this circuit?

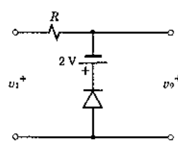


Figure P3.33

P3-34. For $v_i = 10 \sin \omega t$ V, R = 1 kΩ, and V = 4 V in Fig. P3.34, sketch $v_2(t)$.

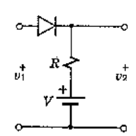


Figure P3.34

P3-35. For $v_i = 10 \sin \omega t$ V, R = 1 kΩ, and V = 4 V in Fig. P3.35, sketch $v_2(t)$.

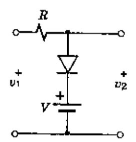


Figure P3.35

P3-36. The periodic voltage v_i of Fig. P3.36 is applied to the input of Fig. P3.33. Show the input signal v_i and the output signal v_o on the same graph.

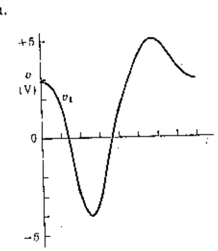


Figure P3.36

P3-37. Repeat Problem P3.36 for the circuit of Fig. P3.37.

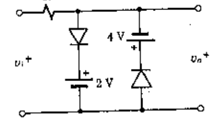


Figure P3.37

P3-38. The periodic voltage v_i of Fig. P3.36 is applied to the input of Fig. P3.38. Show the input signal v_i and the output signal v_o on the same graph.

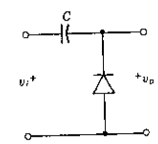


Figure P3.38

P3-39. Repeat Problem 3.38 for the circuit of Fig. P3.39 where V_B = 2 V.

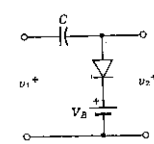


Figure P3.39

P3-40. In Fig. P3.39, $v_i = 10 \sin \omega t$ V and V_B = 4 V. Assuming v_i has been applied a "long" time, plot v_1, v_C, and v_2 on the same time axis.

P3-41. In Fig. P3.41, when voltage $v_i = V_m(0.5 + \sin \omega t)$ V, the high-resistance dc voltmeter VM reads 70 V.

(a) What functions are performed by each section of the circuits?
(b) How is v_3 related to v_1? Show v_1, v_2, and v_3 on the same graph.
(c) Determine V_m and define the function of this instrument.

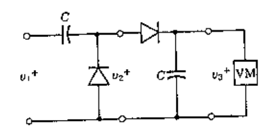


Figure P3.41

P3-42. Devise a circuit using ideal amplifiers to provide an output $v_o = 10 \int v_1 \, dt - 5v_2$.

P3-43. Devise a sweep circuit using an op amp to provide an output voltage proportional to time. Show a reset switch to ensure that v_o = 0 at t = 0.

P3-44. Find the effective and average values of the waveforms of Figs. P3.44a and b.

Answer: (a) effective value = 1.15 V;
(b) effective value = 4.24 A.

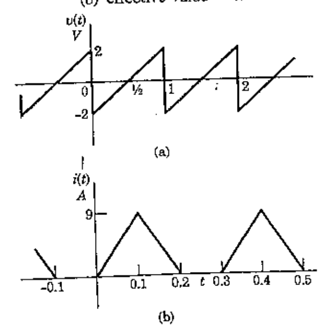


Figure P3.44

P3-45. A regular periodic sinusoidal voltage is given by $v = A \cos \omega t$. Find the effective value and average value for (a) the sinusoid, (b) the half-wave rectified sinusoid, and (c) the full-wave rectified sinusoid.

P3-46. Consider a periodic triangular wave as shown in Fig. P3.46. Find the effective value of (a) the triangular wave, (b) the half-wave rectified triangular wave, and (c) the full-wave rectified triangular wave.

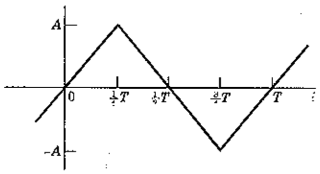


Figure P3.46

P3-47. If $V = 20$ volts and $R_L = 100\ k\Omega$, what is the instantaneous voltage drop across the diode and what is the instantaneous current in R_L? Assume ideal conditions and sketch v_D of the diode and i of the resistor current.

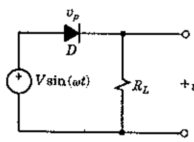


Figure P3.47

P3-48. Given a half-wave rectifier connected to a generator with 100 ohms internal resistance, find the instantaneous output voltage v_o. The forward diode resistance is 200 ohms, and the reverse resistance is 200 kilohms.

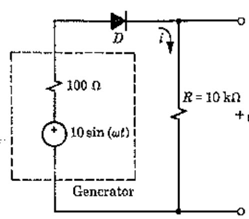


Figure P3.48

P3-49. Assume that the diode in the circuit of Fig. P3.49 is ideal. When the switch is closed at $t = 0$, the capacitor has no initial charge. Sketch at least one cycle of the output voltage v_o beginning at $t = 0$.

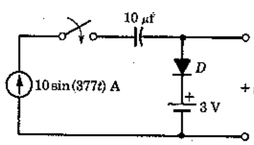


Figure P3.49

ADVANCED PROBLEMS

AP3-1. Demonstrate analytically that the tangent to any exponential function at time t intersects the time axis at $t + \tau$ where τ is the time constant.

AP3-2. The voltage across a 10-μF capacitor is observed on an oscilloscope. In 0.5 s after a 10-V source is removed, the voltage has decayed to 1.35 V. Derive and label a circuit model for this capacitor.

AP3-3. A current $i = 2 \cos 2000t$ A flows through a series combination of $L = 30$ mH and $C = 5$ μF. (C is initially uncharged.)

(a) Determine the total voltage across the combination as $f(t)$.

(b) Repeat part (a) for L increased to 50 mH and interpret this result physically.

AP3-4. A voltage consists of a dc component of magnitude V_0 and a sinusoidal component of effective value V_1; show that the effective value of the combination is $(V_0^2 + V_1^2)^{1/2}$.

AP3-5. The circuit of Fig. AP3.5 is a practical means for obtaining a high dc voltage without using a transformer. Stating any simplifying assumptions, predict voltages V_1 and V_L.

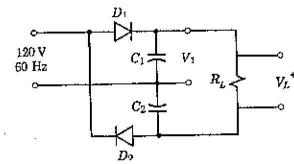


Figure AP3.5

AP3-6. A diode and battery are used in the "voltage regulator" circuit of Fig. AP3.6, where $R_S = 1000\ \Omega$ and $R_L = 2000\ \Omega$. If V_1 increases from 16 to 24 V (a 50% increase), calculate the corresponding variation in load voltage V_L. Is the "regulator" doing its job?

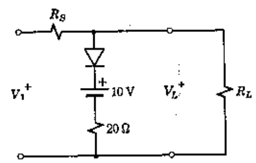


Figure AP3.6

AP3-7. A voltage v_1 varies from -3 to $+6$ V. For a certain purpose, this voltage must be limited to a maximum value of +4.0 V. A diode, a 6-V battery, and assorted resistances are available. Design a suitable circuit.

AP3-8. The output of a flowmeter is $v = Kq$, where q is in cm³/s and $K = 20$ mV-s/cm³. The effective output resistance of the flowmeter is 2000 Ω. Design a circuit that will develop an output voltage $V_o = 10$ V (to trip a relay) after 200 cm³ have passed the metering point.

AP3-9. In Fig. AP3.9, input voltages A and B are restricted to either 0 or +5 V. Tabulate the four possible combinations of A and B and the corresponding values of output voltage V_o. Define in words the output in terms of the inputs. Why is this called an **OR** circuit? Where is it useful?

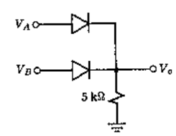


Figure AP3.9

AP3-10. Calculate the exact gain of the circuit of Fig. 3.18.