Engineering 11
Assignment Solutions

(1.) 

$\Sigma i = 0 \Rightarrow \frac{v}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt} = I_0\,\delta(t)$ let $v = \frac{d\lambda}{dt}$

$\frac{d^2\lambda}{dt^2} + \frac{1}{RC}\frac{d\lambda}{dt} + \frac{1}{LC}\lambda = \frac{I_0}{C}\delta(t)$

$i_s(t) = I_0\,\delta(t)$

Homogeneous: set source $= 0$ ; $\lambda_{test} = Ae^{st} \Rightarrow s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$

$R = L = C = 1 \Rightarrow s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ so $\lambda_h = Ae^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t + \phi\right)$ ; $\lambda_p = 0$ (always true for impulses)

IC: $\int_{0^-}^{0^+}\frac{d^2\lambda}{dt^2}dt + \frac{1}{RC}\int_{0^-}^{0^+}\frac{d\lambda}{dt}dt + \frac{1}{LC}\int_{0^-}^{0^+}\lambda\,dt = \frac{I_0}{C}\int_{0^-}^{0^+}\delta(t)\,dt \Rightarrow \left.\frac{d\lambda}{dt}\right|_{0^-}^{0^+} + \frac{1}{RC}\lambda\Big|_{0^-}^{0^+} + 0 = \frac{I_0}{C} \Rightarrow \left.\frac{d\lambda}{dt}\right|_{0^+} + \frac{1}{RC}\lambda(0^+) = \frac{I_0}{C}$

[ $\int_{0^-}^{0^+}\lambda\,dt = 0$ (area) as long as $\lambda(0^+)$ is finite  area $\to 0$ ]

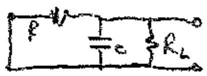
For finite $\frac{d\lambda}{dt}$, $\lambda(0^+) = 0$
$\Rightarrow \left.\frac{d\lambda}{dt}\right|_{t=0^+} = \frac{I_0}{C}$

$\lambda(0^+) = 0 = A\cos\phi \Rightarrow \cos\phi = \pm\pi/2$ (choose $-\pi/2$)

$\left.\frac{d\lambda}{dt}\right|_{t=0^+} = \frac{I_0}{C} = -\frac{1}{2}A\cos\phi - \frac{\sqrt{3}}{2}A\sin\phi \Rightarrow A = \frac{2}{\sqrt{3}}\frac{I_0}{\cancel{C}\,1} \Rightarrow \lambda(t) = \frac{2}{\sqrt{3}}I_0 e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t - \pi/2\right)u(t)$

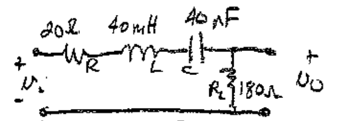
$v(t) = \frac{d\lambda}{dt} = -\frac{1}{\sqrt{3}}I_0 e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t - \pi/2\right)u(t) - \frac{4}{3}I_0 e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{2}\right)u(t) + \underbrace{\frac{2}{\sqrt{3}}I_0 e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{2}\right)\delta(t)}_{=0}$

(2. 14.6) a. $f_c = \frac{\omega_c}{2\pi} = \boxed{7.958\text{ kHz}}$ b. $\frac{1}{RC} = 50(10^3) \Rightarrow \boxed{R = 40\ \Omega}$

c.  $Z(s) = \frac{R_{eq}}{s + R_{eq}C}$, $R_{eq} = \frac{RR_L}{R + R_L} \Rightarrow s = -\frac{1}{R_{eq}C} = -\omega_c \Rightarrow R_L = 800\ \Omega$

d. For $\omega \to 0$, $Z_C = \frac{1}{j\omega C} \to \infty \Rightarrow$  $\left.\frac{V_0}{V_i}\right|_{\omega\to 0} = \frac{R_L}{R + R_L} = .9524$

(3. 14.17) a. $\omega_0 = \sqrt{\frac{1}{LC}} = 25$ krad/s $\Rightarrow f_0 = \frac{\omega_0}{2\pi} = 3.979$ kHz



$\frac{V_0}{V_i} = \frac{R_L}{sL + (R + R_L) + \frac{1}{sC}}$

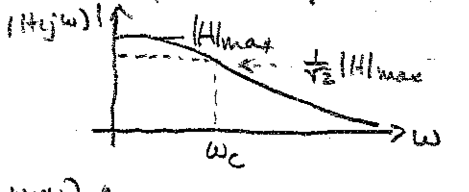
$\frac{V_0}{V_i} = \frac{R_L}{R + R_L + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R_L/(R + R_L)}{1 + j\left(\frac{1}{R + R_L}\right)\left(\omega L - \frac{1}{\omega C}\right)} = \frac{\frac{R_L}{(R + R_L)}}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$

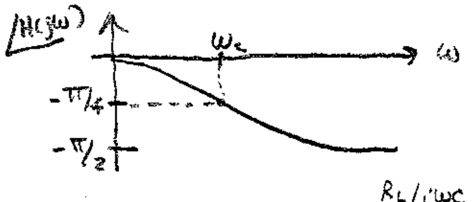
b. $Q = \frac{\omega_0 L}{R + R_L} = 5$

c. $\omega_{c1} = \omega_0\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = 22.6$ krad/s

d. $\omega_{c2} = \omega_0\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right] = \boxed{27.6\text{ krad/s}}$

(4.) Plot amplitude & phase of 14.6



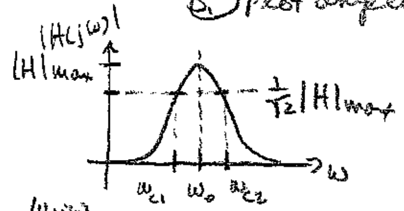



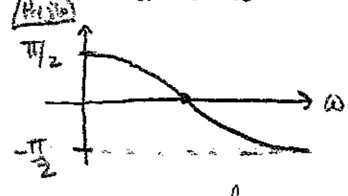

$H(j\omega) = \frac{R_L \| Z_C}{R + R_L \| Z_C} = \frac{\frac{R_L/j\omega C}{R_L + 1/j\omega C}}{R + \frac{R_L/j\omega C}{R_L + 1/j\omega C}}$

$= \frac{\frac{R_L}{R + R_L}}{1 + j\omega\left(\frac{RR_LC}{R + R_L}\right)} \Big/ -\tan^{-1}\omega\left(\frac{RR_LC}{R + R_L}\right)$

$|H|_{max} = \frac{R_L}{R + R_L}$

(5.) Plot amplitude & phase of 14.17




$H(j\omega) = \frac{R_L}{R + R_L}\,\frac{j\omega}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$

$= K\sqrt{\frac{1}{1 + \left[Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]^2}}\ \Big/ \frac{\pi}{2} - \tan^{-1}\left[Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$

$|H|_{max} = K = \frac{R_L}{R + R_L}$