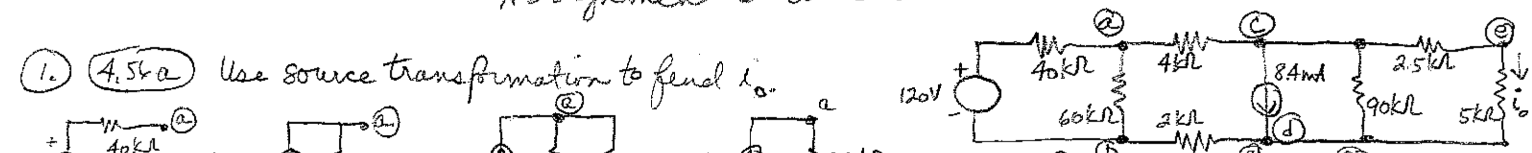
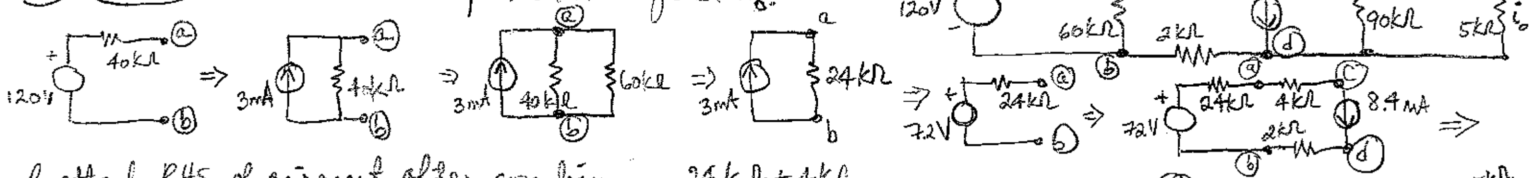
4.56a, 4.61, 4.69,
4.75, 4.85, 4.90, 4.94

Engineering 11
Electrical Circuit Analysis
Assignment 3 Solutions

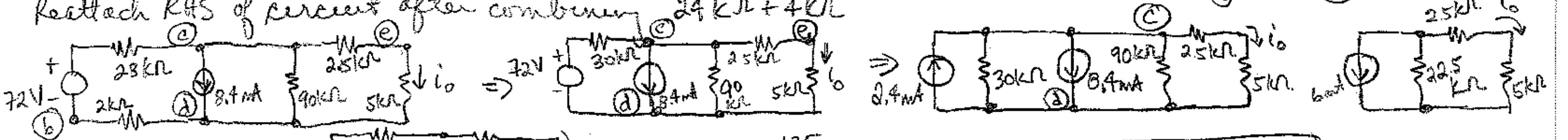
2005
LAM

(1.) (4.56a) Use source transformation to find i_o.

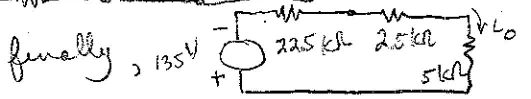




Reattach RHS of circuit after combining $24\,k\Omega + 4\,k\Omega$

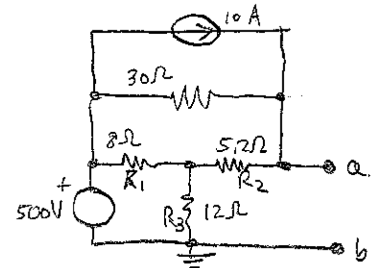


& finally,



$$i_o = \frac{135}{(22.5+2.5+5)k} = \frac{135}{30k} = \boxed{4.5\text{ mA} = i_o}$$

(2) (4.61) Find the Thevenin equivalent @ terminals a b.

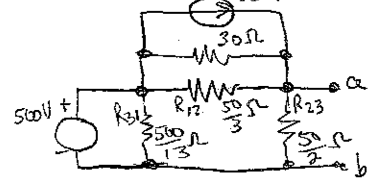


Use Y → Δ transformation.

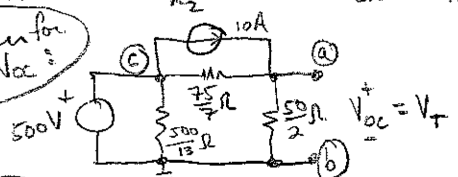
$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{200}{12} = \frac{50}{3}\,\Omega$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{200}{8} = \frac{50}{2}\,\Omega$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{200}{5.2} = \frac{500}{13}\,\Omega$$



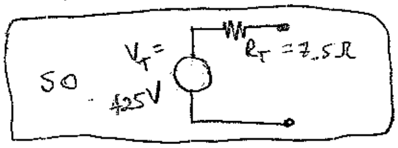
Then for $V_T = V_{oc}$:



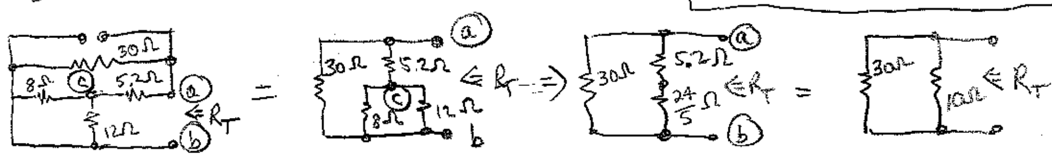
$V_c = 500\,V$
$V_b = 0\,V$
$V_a = V_{oc}$ = unknown

KCL:
$$\sum_a i = 0 = 10 + \frac{500 - V_a}{(75/7)} - \frac{V_a}{(50/2)} = 0 \Rightarrow V_a = \boxed{V_{oc} = 425\,V = V_T}$$

So



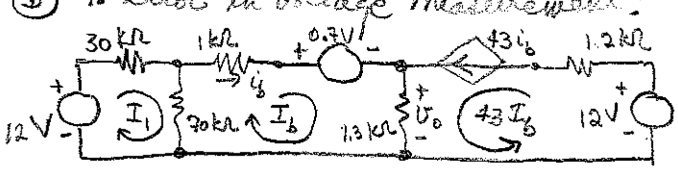
For R_T: remove sources, look into a b & find equiv. R



$$R_T = 10 \| 30 = \frac{10(30)}{10+30} = \boxed{7.5\,\Omega}$$

(3.) (4.69) (a) VM reads 5.5 V. What is resistance of VM? (b) % error in voltage measurement?

(VM places R_m in parallel w/ 1.3 kΩ resistor.)



Actual voltage: (note i_b = loop current I_b)

loop 1: $\sum i = 0 = 12 - 30k\,I_1 - 70k(I_1 - I_b) = 0$

loop 2: $\sum i = 0 = -1k\,I_b - 0.7 - 1.3k(I_b + 43I_b) - 70k(I_b - I_1)$

$$\Rightarrow \begin{array}{l} 100k\,I_1 - 70k\,I_b = 12 \\ 70k\,I_1 - [71k + 44(1.3k)]I_b = 0.7 \end{array} \Rightarrow \text{using MATLAB} \Rightarrow \begin{bmatrix} I_1 \\ I_b \end{bmatrix} = [\quad]$$

$v_o = 1.3k(44 I_b) =$

⇒ & (b) $\text{Error} = \frac{5.5611 - 5.5}{5.5611} \approx 1.1\%$

To find R_m, replace 1.3k resistor by $R_{eq} = 1.3k \| R_m$.

so
$$\begin{array}{l} 100k\,I_1 - 70k\,I_b = 12 \\ 70k\,I_1 - [71k + 44(R_{eq})]I_b = 0.7 \end{array} \Rightarrow \text{using MATLAB} \Rightarrow \begin{bmatrix} I_1 \\ I_b \end{bmatrix} = [\quad]$$

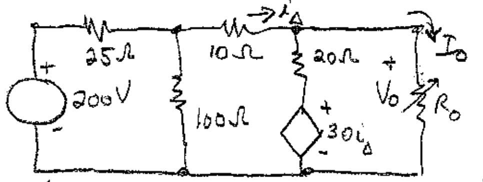
and we also need $v_o = 5.5 = 44 R_{eq} I_b$; sub into for R_{eq}.

so $R_{eq} = \frac{5.5}{44 I_b} = 1.25\,k\Omega$

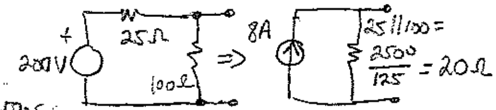
Then use $R_{eq} = \frac{1.3k R_m}{1.3k + R_m}$

$\Rightarrow \boxed{R_m = 32.5\,k\Omega}$

(4. 4.75) Adjust R_o until $P_{R_o} = 250$ W.

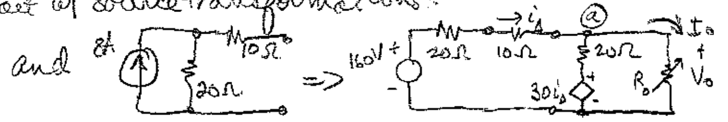


$P_{R_o} = \frac{V_o^2}{R_o} = I_o^2 R_o$



Start w/ source transformations:

and



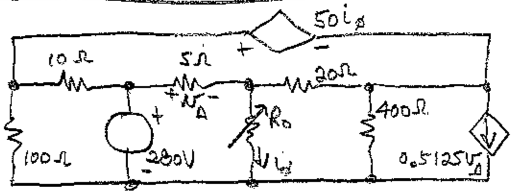
$\sum_a i = 0 = \frac{160 - V_o}{30} - \frac{V_o - 30 i_\Delta}{20} - \frac{V_o}{R_o} = 0$; $i_\Delta = \frac{160 - V_o}{30}$;

$R_o = \frac{V_o^2}{P_{R_o}} = \frac{V_o^2}{250}$

So $\frac{160 - V_o}{30} - \frac{V_o - (160 - V_o)}{20} - \frac{V_o}{(V_o^2/250)} = 0$

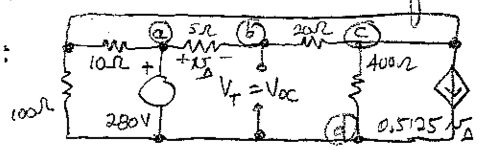
solving quadratic, $V_o = 75, 25$ => $R_o = V_o^2/250 = 22.5\Omega, 2.5\Omega$

(5. 4.85) Adjust R_o for maximum power transfer.



(a) Find R_o. Find Thevenin equivalent of all except R_o

V_T :



when R_o is removed, $i_\phi \to 0$

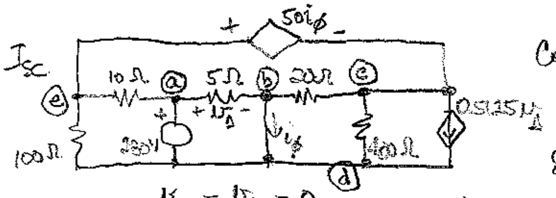
=> voltage source across the top of the circuit → S.C.

$V_a = 280$ V ; collapse nodes abc into supernode.

$\sum_c i = 0 = -\frac{v_c}{100} - \frac{v_c - 280}{10} - \frac{v_c}{400} - \frac{v_c - 280}{25} - 0.5125 v_\Delta = 0$; $v_\Delta = \frac{5}{25}(280 - v_c)$ => $v_c = 210$ V, $v_\Delta = 14$ V.

so $V_T = V_{oc} = 280 - v_\Delta = 266$ V

I_{sc}



$v_b = v_d = 0$

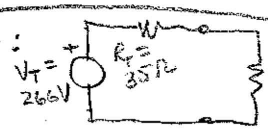
Consider ce as a supernode.

$\frac{v_e}{100} + \frac{v_e - 280}{10} + 0.5125 v_\Delta + \frac{v_c}{400} + \frac{v_c}{20} = 0$; Note $v_\Delta = 280$

Supernode eq: $v_c + 50 i_\phi = v_e$, and $i_\phi = \frac{280 - 0}{5} + \frac{v_c - 0}{20}$

Using MATLAB, $v_c = -968$ V and $v_e = -588$ V => $i_\phi = I_{sc} = 7.6$ A

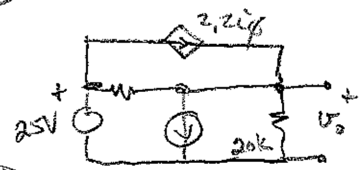
Thevenin equiv:

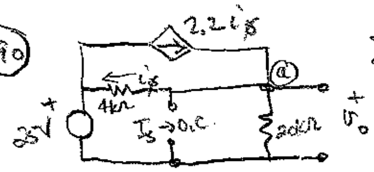


$R_T = \frac{V_{oc}}{I_{sc}} = 35\Omega$

$R_o = 35\Omega$ for max power transfer ($R_o = R_T$)

(6. 4.90)

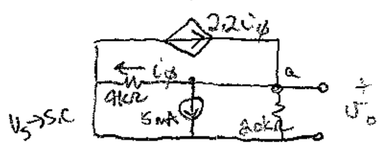




$\sum_a i = 0 = 2.2 i_\phi - i_\phi - \frac{v_o}{20k}$; $i_\phi = \frac{v_o - 25}{4k}$

=> $v_o|_{V_s} = 30$ V

$\therefore v_o|_{total} = 30 + 20 = 50$ V



$\sum_a i = 0 = 2.2 i_\phi - i_\phi - 5m - \frac{v_o}{20k} = 0$

$i_\phi = \frac{v_o}{4k}$ => $v_o|_{I_s} = 20$ V

(7. 4.97) $V_{no\ load} = 75$ V ; $V|_{20\Omega} = 60$ V

no load => $R_L \to \infty$ (O.C.) => $V_L = V_T = 75$ V

(a) Thevenin Model:

$V_L = 60 = \frac{R_L}{R_T + R_L} V_T = \frac{20}{R_T + 20}(60)$ => $R_T = 5\Omega$

(b) $V_o = \frac{R_L}{R_T + R_L} V_T$ => $V_T = (1 + \frac{R_T}{R_L}) V_o$ or $\frac{R_T}{R_L} = \frac{V_T}{V_o} - 1$ => $R_T = (\frac{V_T}{V_o} - 1) R_L$