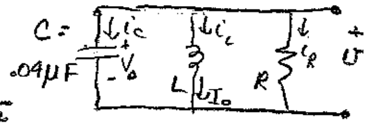
8.7, 8.12, 8.23, 8.51, 8.56

Engineering 11
Assignment 7 Solutions

2005
LA

1. 8.7 $v(t) = 100\,e^{-20000t}\,[\cos 15{,}000t - 2\sin 15000t]\ \text{V};\ t \geq 0$



b. Find R $\quad Y(s) = \frac{1}{R} + \frac{1}{sL} + sC = \frac{s^2RLC + sL + R}{sLR} = \frac{1}{Z(s)}$

so Char eq is $s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC} = 0 \Rightarrow s = -\frac{1}{2RC} \pm j\sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2} = -\alpha \pm j\omega = -20000 \pm j15000$

$\frac{1}{2RC} = 20000 \Rightarrow \boxed{R = 625\,\Omega}$

a. $\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2 = (15000)^2$; use R from above $\Rightarrow \boxed{L = 40\,\text{mH}}$

c. Find $\boxed{V_0 = v(0) = 100\,\text{V}}$

d. Find $I_0 = i_L(0) = -i_R(0) - i_C(0) = -\frac{v(0)}{R} - C\frac{dv}{dt}\Big|_{t=0} = \frac{-100}{625} - 0.04(100)[-20000 - 30000] = \boxed{40\,\text{mA}}$

e. $i_L(t) = -i_R(t) - i_C(t) = -\frac{v(t)}{R} - C\frac{dv(t)}{dt} = \boxed{e^{-20000t}\,[40\cos 15000t + 220\sin 15000t]\,\text{A};\ t \geq 0}$

2. 8.12 Same circuit as in 8.7. $L = 12.5\,H$, $C = 3.2\,nF$, R gives critical damping. $V_0 = 100\,V$, $I_0 = 6.4\,mA$.

a. Find R. (Use Z(s) from above.) For critical damping, $\frac{1}{LC} = \left(\frac{1}{2RC}\right)^2 \Rightarrow \boxed{R = 31.25\,k\Omega}$

b. Find $v(t) = A_1 e^{-t/2RC} + A_2 t e^{-t/2RC} = A_1 e^{-5000t} + A_2 t e^{-5000t}$

$v(0) = 100 = A_1 \qquad \frac{dv}{dt}\Big|_{t=0} = \frac{i_C(0)}{C} = -\frac{(i_R(0) + i_L(0))}{C} = -\frac{v(0)}{RC} - \frac{I_0}{C} = -3(10^6)\ \text{V/s}$

so $\frac{dv}{dt}\Big|_{t=0} = -3(10^6) = -5000A_1 + A_2 \Rightarrow A_2 = -2.5(10^6)$

$\boxed{v(t) = 100e^{-5000t} - 2.5(10^6)te^{-5000t}\ \text{V};\ t \geq 0}$

c. Find v(t) when $i_C(t) = 0$. $\quad i_C = C\frac{dv}{dt}$; $\frac{dv}{dt} = -5(10^5)e^{-5000t} - 2.5(10^6)e^{-5000t} - 5(2.5)(10^9)te^{-5000t}$

$\frac{dv}{dt} = 0$ when $-5(10^5) - 2.5(10^6) - 12.5(10^9)t = 0$ or @ $t = 240\,\mu s$.

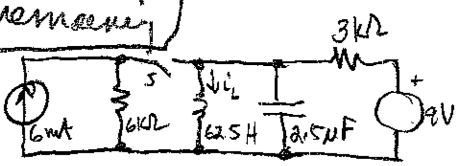
$\boxed{v(240\,\mu s) = -150.6\,V}$

d. % of energy remaining @ $t = 240\,\mu s$ $\qquad \frac{w(240)}{w(0)} = \frac{\frac{1}{2}Li_L^2(240\mu) + \frac{1}{2}Cv_C^2(240\mu)}{\frac{1}{2}Li_L^2(0) + \frac{1}{2}Cv_C^2(0)}$

$v_C = v$ so plug in $t = 0$ & $t = 240\,\mu s$ to above eq for v

@ $t = 240\,\mu s$, $i_C = 0$ so $i_L = -i_R = -\frac{v(240\mu)}{R}$, & $i_L(0)$ is given in problem statement: 6.4mA

Plugging all in $\Rightarrow \frac{145\mu + 36.3\mu}{256\mu + 16\mu} \Rightarrow \sim\boxed{66.7\%\ \text{remaining}}$



3. 8.23 S open for a long time; closes @ $t = 0$. Find $i_L(t)$

@ $t = 0^-$, $v_C(0^-) = 0$, $i_L(0^-) = 3\,mA$. $\qquad$ @ $t \to \infty$, $i_L(t \to \infty) = (6 + 3)\,mA = 9\,mA = I_p$

To find i_L, redraw circuit (remove sources) & find → Z(s) =>

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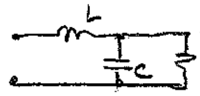


$R = 3\|6 = 2\,k\Omega$

$Z(s) = sL + \frac{R/sC}{R + 1/sC} = \frac{s^2RLC + sL + R}{sRC + 1}$

For homogeneous current, $s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC} = 0 \xrightarrow{\text{quad. eq}} s = -100 \pm 60$ (both real)

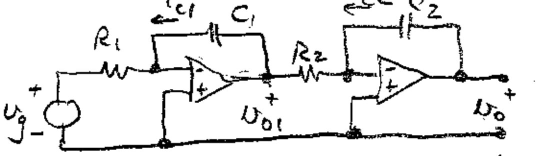
$i_L(t) = I_p + A_1e^{-40t} + A_2e^{-160t}$ (I_p as above)

$i_L(0) = 3m = 9m + A_1 + A_2$; $\quad \frac{di_L}{dt} = \frac{v_L(0^+)}{L} = \frac{v_C(0^+)}{L} = \frac{v_C(0^-)}{L} = 0 = -40A_1 - 160A_2$ $\quad$ Solve simultaneously $\Rightarrow$

$A_1 = -8\,mA$, $A_2 = 2\,mA$ & $\boxed{i_L(t) = 9 - 8e^{-40t} + 2e^{-160t}\ \text{mA};\ t \geq 0}$

4.8.51 (a) $\sum i = C_1 \frac{d}{dt}(v_{01}-0) + \frac{v_g - 0}{R_1} = 0$ ①

$\sum i = C_2 \frac{d}{dt}(v_0 - 0) + \frac{v_{01}-0}{R_2} = 0$ ② $\frac{d}{dt}$ of ② & sub into ①



$$\frac{d^2 v_0}{dt^2} = \frac{1}{R_1 C_1 R_2 C_2} v_g = 250\, v_g$$

$\boxed{0 \le t < 0.05\,s}$ $v_g = 80\,mV \Rightarrow \frac{d^2 v_0}{dt^2} = 20 \xrightarrow{\int \text{twice}} v_0(t) = 10t^2 + \cancel{C_1 t} + \cancel{C_2}$ $\to 0$ for no initial energy storage. $C_2 = 0$ gives $v_0(0) = 0$; $C_1 = 0$ gives $\frac{dv_0}{dt}\Big|_{t=0} = \frac{i_C}{C} = 0$.

eq ① $\Rightarrow \frac{dv_{01}}{dt} = \frac{-v_g}{R_1 C_1} = -20 v_g = -1.6 \xrightarrow{\int} v_{01}(t) = -1.6t + C_3$

$\boxed{0.5 < t < t_{sat}}$ $\frac{d^2 v_0}{dt^2} = 250 v_g = -10 \Rightarrow v_0 = -5t^2 + C_1' t + C_2'$

$C_2' = v_0(0.05) = -3.75\,V$ since $v_0 = v_{C2}$ can't change

$\frac{dv_{01}}{dt} = -20 v_g = 0.8 \Rightarrow v_{01} = 0.8t + C_3'$

$C_3' = v_{01}(0.05) = -1.2\,V$ since $v_{01} = v_{C1}$ can't change

still need C_1'. Note $\frac{dv_0}{dt}\Big|_{t=0.05} = -10t\Big|_{0.05} + C_1' = \frac{i_{C2}(0^+)}{C_2} = \frac{0 - v_{01}(0.05)}{R_2} = 15$

So $0 \le t < 0.05\,s$: $v_0(t) = 10t^2\ V$, $v_{01}(t) = -1.6t\ V$

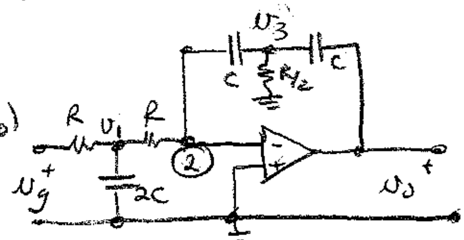
$0.05\,s < t < t_{sat}$: $v_0(t) = -5t^2 + 15t - 3.75\ V$; $v_{01}(t) = 0.8t - 1.2\,V$

(b) $v_0|_{max} = -12.5\,V = -5t_{sat}^2 + 15t_{sat} - 3.75 \Rightarrow$ solve quadratic for $\boxed{t_{sat} = 3.5\,s}$

Check that v_{01} is not saturated : $v_{01}(t_{sat}) = 1.6\,V$ ✓ ok.

5. 8.56 $\sum i = 0 = \frac{v_g - v_1}{R} - 2C\frac{dv_1}{dt} - \frac{v_1}{R} = 0$

$\sum_2 i = 0 = \frac{v_1}{R} + C\frac{dv_3}{dt}$ $\quad \sum_3 i = 0 = C\frac{d}{dt}(-v_3) - \frac{v_3}{R/2} - C\frac{d}{dt}(v_3 - v_0)$



① $\frac{dv_1}{dt} + \frac{v_1}{RC} = \frac{v_g}{2RC}$

② $\frac{dv_3}{dt} = -\frac{1}{RC} v_1 \Rightarrow v_1 = -RC\frac{dv_3}{dt}$

③ $\frac{dv_3}{dt} + \frac{v_3}{RC} = -\frac{1}{2}\frac{dv_0}{dt}$

Plug ② into ① for v_1. Take $\frac{d}{dt}$② to find $\frac{dv_1}{dt}$ & plug into ①. Take $\frac{d}{dt}$③

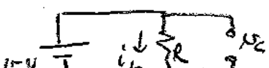
eqs 1 & 2 become $-RC\frac{d^2 v_3}{dt^2} - \frac{dv_3}{dt} = \frac{v_g}{2RC} \Rightarrow \frac{d^2 v_3}{dt^2} + \frac{1}{RC}\frac{dv_3}{dt} = -\frac{v_g}{2R^2C^2}$

eq 3 becomes $\frac{d^2 v_3}{dt^2} + \frac{1}{RC}\frac{dv_3}{dt} = \frac{1}{2}\frac{d^2 v_0}{dt^2}$ (LHS are the same)

so $\frac{d^2 v_0}{dt^2} = -\frac{v_g}{R^2C^2}$

b. The same when $R_1C_1 = R_2C_2 = RC$

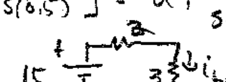
c. Integrate 2x using just 1 op amp

6. a. long time : 

$v_C = 15\,V$, $i_L = \frac{15}{3} = 5\,A$

b. $t = 0^+$ $i_L = 5\,A$, $v_C = 15\,V$, $i = 0$ since all of v drops across C & none across left R), $\boxed{i_C = -i_L = -5\,A}$

c, d. $Z(s) = 2 + \left[(s+3) \,\|\, \frac{1}{s(0.5)}\right] = 2 + \frac{(s+3)/0.5s}{s+3+\frac{1}{0.5s}} = 2\frac{s^2+4s+5}{s^2+3s+2}$; For i use zeros $\Rightarrow s = -2 \pm j$

$i_{Lp} = \frac{15}{3+2} = 3\,A$ 

$i_L = i_{Lh} + i_{Lp} = Ae^{-2t}\cos(t + \phi) + 3$

$i_L(0) = 0 = A\cos\phi + 3$

$\frac{di_L}{dt}\Big|_{t=0} = \frac{d}{dt}\left(\frac{15 - v_C}{2}\right) = -\frac{1}{2}\frac{dv_C}{dt} = -\frac{i_C}{2C} = 5\,A/s$

$= -2A\cos\phi - A\sin\phi$

$\boxed{i(t) = 3 - 3.16\,e^{-2t}\cos(t - 18.4^\circ)\,A \quad t \ge 0}$