

$$v_1 = v_a + v_i = v_a \quad \text{and} \quad v_a = \frac{R_1}{R_1 + R_F} v_o \quad (3-23)$$

where R_1 and R_F constitute a voltage divider across the output voltage. The gain of the *noninverting amplifier* circuit is

$$A_F = \frac{v_o}{v_1} = \frac{R_1 + R_F}{R_1} \quad (3-24)$$

Here again the gain is determined by the feedback network elements R_1 and R_F . For the noninverting circuit, however, the gain is positive and equal to or greater than unity.

Example 8

An op amp is used in the circuit of Fig. 3.23, where input signal v_s is a function of time (a sinusoid or a combination of sinusoids). Derive an expression for $i_o(t)$.

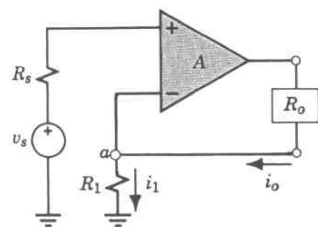


Figure 3.23 Voltage-current converter.

Because no current flows into an ideal op amp, there is no drop across R_s , and voltage $v_s(t)$ appears between the + terminal and ground.

Because there is no input voltage across the input terminals of an ideal op amp, $v_a = v_s = i_1 R_1$. But current i_1 must be supplied by the feedback current i_o . Hence

$$i_o(t) = i_1(t) = \frac{1}{R_1} v_s(t)$$

This noninverting circuit is serving as a voltage-to-current converter. The output current is proportional to the input voltage and independent of R_s and R_o .

Exercise 3-9

An ideal amplifier is used in the circuit of Fig. 3-22 with $R_1 = 5 \text{ k}\Omega$ and $R_F = 500 \text{ k}\Omega$.

- For an input voltage $v_1 = 0.01 \text{ V}$, what is v_a ? What is i_i ?
- Considering the voltage divider R_1 - R_F , what must v_o be?
- Calculate the voltage gain v_o/v_i by Eq. 3-24.

Answers: (a) 0.01 V, 0; (b) 1.01 V; (c) 101.

3-7

IDEAL DIODES

A useful addition to our repertoire of electronic circuit elements is the *diode* or *rectifier*. A diode is a two-terminal device that acts as a switch; it permits current to flow readily in one direction but it tends to prevent the flow of current in the

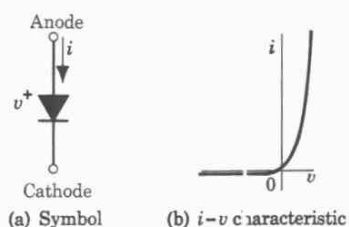


Figure 3.24 A real diode.

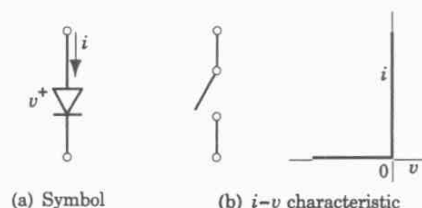


Figure 3.25 An ideal diode.

other direction. The direction of easy current flow is indicated by the arrowhead in the symbol of Fig. 3.24a. This is a nonlinear circuit element, and it is useful because it is *not* linear. However, the analysis of circuits containing diodes may be complicated because network theorems based on linearity cannot be used. Figure 3.24b shows the i - v characteristic of a semiconductor diode consisting of a junction of dissimilar materials that we will study later. Just as we use ideal models to represent real R , L , and C components, so we can approximate a real diode by the ideal characteristics shown in Fig. 3.25b. When the anode is positive with respect to the cathode, that is, when voltage v is positive, the “switch” is closed and unlimited current i can flow with no voltage drop. In contrast, when the cathode is positive with respect to the anode, the “switch” is open and no current flows even for large negative values of voltage v .

RECTIFIERS

The nonlinear characteristic of a diode is used to convert alternating current into unidirectional, but pulsating, current in the process called *rectification*. The pulsations are removed in a frequency-selective circuit called a *filter*. Rectifier circuits employ one, two, or four diodes to provide various degrees of rectifying effectiveness. Filter circuits use the energy-storage capabilities of inductors and capacitors to smooth out the pulsations and to provide a steady output current. A combination of ac source, rectifier, and filter is called a *power supply*.

HALF-WAVE RECTIFIER. Ideally, a diode should conduct current freely in the forward direction and prevent current flow in the reverse direction. Practical diodes only approach the ideal. Semiconductor diodes, for example, present a small but appreciable voltage drop in the forward direction and permit a finite current to flow in the reverse direction. For most calculations, the reverse current flow is negligibly small and the forward voltage drop can be neglected with little error.

A practical circuit for *half-wave rectification* is shown in Fig. 3.26a. A *transformer* supplied from 120-V, 60-Hz house current provides the desired operating voltage, which is applied to a series combination of diode and load resistance R_L . (The transformer, consisting of two multturn coils wound on a common iron core, provides a voltage “step-down” in direct proportion to the turn ratio.) For approximate analysis, the actual diode is represented by an ideal diode; the internal

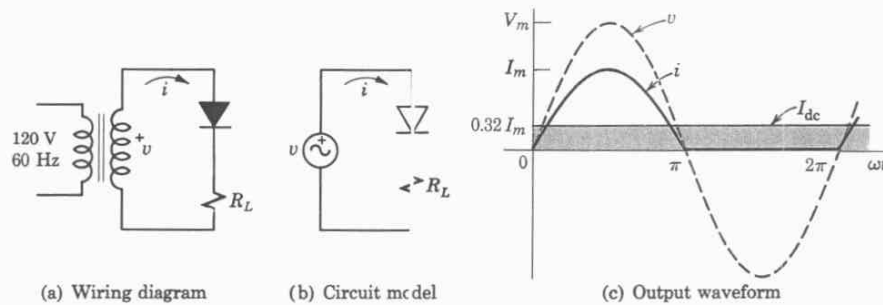


Figure 3.26 A half-wave rectifier.

resistance of the transformer is neglected. For $V = V_m \sin \omega t$, the resulting current is

$$\begin{cases} i = \frac{v}{R_L} = \frac{V_m \sin \omega t}{R_L} & \text{for } 0 \leq \omega t \leq \pi \\ i = 0 & \text{for } \pi \leq \omega t \leq 2\pi \end{cases} \quad (3-25)$$

as shown in Fig. 3.26c.

$$\begin{aligned} I_{dc} &= \frac{1}{2\pi} \int_0^{2\pi} i \, d(\omega t) = \frac{1}{2\pi} \int_0^{\pi} \frac{V_m \sin \omega t}{R_L} \, d(\omega t) + 0 \\ &= \frac{1}{2\pi} \frac{V_m}{R_L} [-\cos \omega t]_0^{\pi} = \frac{V_m}{\pi R_L} = \frac{I_m}{\pi} \end{aligned} \quad (3-26)$$

The current through the load resistance consists of half-sinewaves, and the dc component is approximately 30% of the maximum value.

Example 9

An ideal diode is connected in the circuit of Fig. 3.27. For $v = 170 \sin \omega t$ V, predict the current through $R = 5 \text{ k}\Omega$.

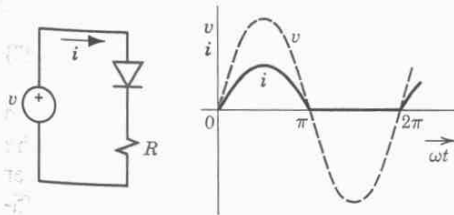


Figure 3.27 Application of diode.

For $0 < \omega t < \pi$, v is positive, the diode switch is closed, and

$$i(t) = \frac{v}{R} = \frac{170 \sin \omega t}{5000} = 34 \sin \omega t \text{ mA}$$

For $\pi < \omega t < 2\pi$, v is negative, the diode switch is open, and current $i = 0$. The resulting current is shown by the solid line. The sinusoidal voltage wave has been *rectified*.

The dc component of the rectified current is (by Eq. 3-26)

$$I_{dc} = \frac{I_m}{\pi} = \frac{34}{\pi} = 10.8 \text{ mA}$$

Exercise 3-10

Find the output voltage, v_o , for the circuit of Fig. E3.10.

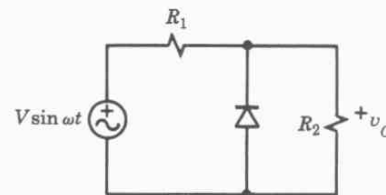
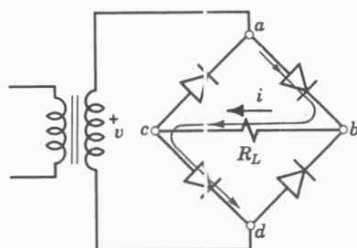


Figure E3.10

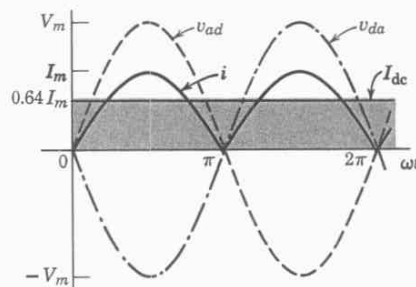
Answer:

$$v_o = \begin{cases} \frac{R_2}{R_1 + R_2} V \sin \omega t & \sin \omega t \geq 0 \\ 0 & \sin \omega t \leq 0 \end{cases}$$

FULL-WAVE RECTIFIER. The bridge rectifier circuit of Fig. 3.28 provides a greater dc value from the same transformer voltage. When the transformer voltage $v = v_{ad}$ is positive, the current flow is along path $abcd$ as shown and a half-sinewave of current results. When the applied voltage reverses, the voltage $v_{da} = -v_{ad}$ is positive and the current flow is along path $dbca$. The current through the



(a) Circuit



(b) Waveforms

Figure 3.28 A full-wave bridge rectifier.

load resistance is always in the same direction, and the dc component is twice as large as in the half-wave rectifier or

$$I_{dc} = \frac{2}{\pi} \frac{V_m}{R_L} = \frac{2I_m}{\pi} \quad (3-27)$$

The bridge circuit is disadvantageous because four diodes are required, and two diodes and their power-dissipating voltage drops are always in series with the load. The full-wave rectifier circuit of Fig. 3.29 uses a more expensive transformer to produce the same result with only two diodes and with higher operating efficiency. The second output winding on the transformer provides a voltage v_2 that is 180° out of phase with v_1 ; such a *center-tapped* winding serves as a *phase inverter*. While v_1 is positive, current i_1 is supplied through diode 1; while v_1 is negative, no

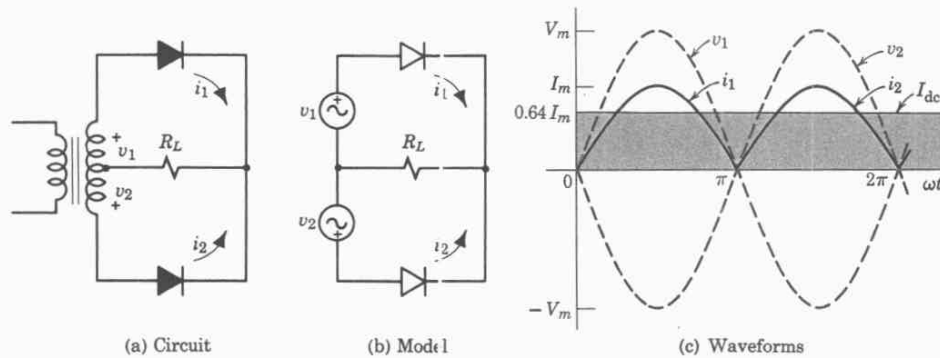


Figure 3.29 A full-wave rectifier with phase inverter.

current flows through diode 1 but v_2 is positive and, therefore, current i_2 is supplied through diode 2. The current through the load resistance is $i_1 + i_2$ and $I_{dc} = 2I_m/\pi$ as in the bridge circuit.

FILTERS

The desired result of rectification is direct current, but the output currents of the rectifier circuits described obviously contain large alternating components along with the dc component. Using full-wave instead of half-wave rectification reduces the ac component, but the remaining *ripple voltage* (voltage variation) across R_L is still unsatisfactory for most electronic applications which require a nearly steady voltage. An *electrical filter* that passes the dc component while rejecting the ac component will provide the desired direct current.

CAPACITOR FILTER. The ripple voltage can be greatly reduced by a filter consisting of a capacitor shunted across the load resistor. The capacitor can be thought of as a "tank" that stores charge during the period when the diode is conducting and releases charge to the load during the nonconducting period.

If the diode is nearly ideal and if the steady state has been reached, the operation is as shown in Fig. 3.30. At time $t = 0$, the source voltage v is zero but

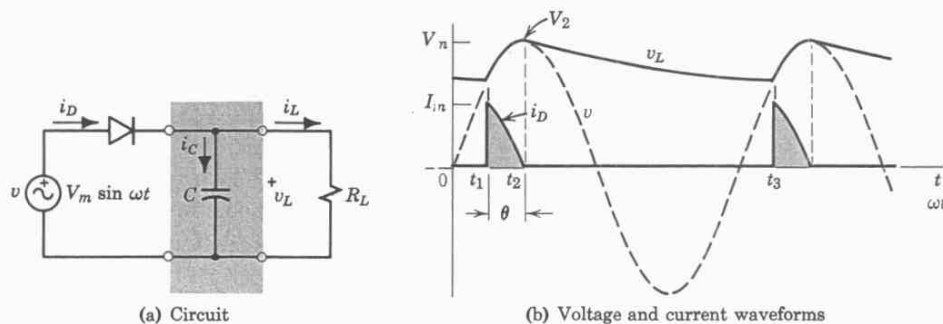


Figure 3.30 A capacitor filter.

the load voltage $v_L = v_C$ is appreciable because the previously charged capacitor is discharging through the load. At $t = t_1$, the increasing supply voltage v slightly exceeds v_L and the diode conducts. The diode current i_D rises abruptly to satisfy the relation $i_C = C dv/dt$ and then decreases to zero; the diode switches off when v drops below v_L . During the charging period, $t_1 < t < t_2$,

$$v_L = V_m \sin \omega t \quad (3-28)$$

During the discharging period, $t_2 < t < t_3$, the capacitor voltage decays exponentially so that

$$v_L = V_2 e^{-(t-t_2)/R_L C} \quad (3-29)$$

with a time constant defined by the RC circuit. At time t_3 , the supply voltage again exceeds the load voltage and the cycle repeats.

The load current i_L is directly proportional to load voltage v_L . Because i_L never goes to zero, the average value or dc component is relatively large as compared to the half-wave rectifier alone and the ac component is correspondingly lower. The ripple voltage is greatly reduced by the use of the capacitor.

CAPACITOR FILTER—APPROXIMATE ANALYSIS. If the time constant $R_L C$ is large compared to the period T of the supply voltage, the decay in voltage $v_C = v_L$ will be small, and the ripple voltage $V_r = \Delta v_C$ will be small. The magnitude of the ripple can be estimated by assuming that V_r is small, the charging interval $t_2 - t_1$ is small, and v_C is nearly constant. Under this assumption, all the load current is supplied by the capacitor, and the charge transferred to the load is

$$\Delta q = I_{dc} T = C \Delta v_C = C V_r \quad (3-30)$$

Solving for the ripple voltage,

$$V_r = \frac{I_{dc} T}{C} = \frac{I_{dc}}{fC} = \frac{V_{dc}}{fR_L C} \quad (3-31)$$

This relation holds for a half-wave rectifier; a similar equation can be derived for a full-wave rectifier. By using this approximation, the performance of an existing filter can be predicted or a filter can be designed to meet specifications as in Example 10.

Example 10

A load ($R_L = 3330 \Omega$) is to be supplied with 50 V at 15 mA with a ripple voltage no more than 1% of the dc voltage. Design a rectifier-filter to meet these specifications.

Assuming a 120-V, 60-Hz supply and the half-wave rectifier with capacitor filter of Fig. 3.30a, Eq. 3-31 is applicable. Solving for C ,

$$C = \frac{V_{dc}/V_r}{fR_L} = \frac{100}{60 \times 3330} = 500 \mu\text{F}$$

For a peak value of 50 V, the rms value of transformer secondary voltage should be $50/\sqrt{2} = 35.4$ V. The transformer turn ratio should be

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{120}{35.4} = 3.39$$

Exercise 3-11

Redraw Fig. 3.30b to show the load voltage V_L and the diode current i_D for a full-wave rectifier-capacitor filter supply. For a full-wave rectifier:

- How many current pulses are there per second?
- Derive the equation for V_r for this case.
- What size capacitor is needed to meet the specifications of Example 10?

Answers: (a) $2f$; (b) $V_r = V_{dc}/2fR_LC$; (c) $250\ \mu\text{F}$.

3-8

WAVESHAPING CIRCUITS

A major virtue of electronic circuits is the ease, speed, and precision with which voltage and current waveforms can be controlled. Some of the basic waveshaping functions are illustrated by a radar pulse-train generator. The word *radar* stands

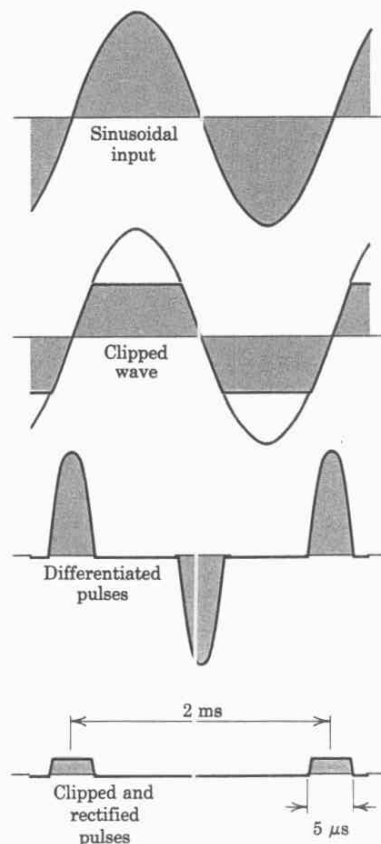


Figure 3.31 Generation of a train of timing pulses.

for radio detection and ranging. A very short burst of high-intensity radiation is transmitted in a given direction; a return echo indicates the presence, distance, direction, and speed of a reflecting object. The operation of a radar system requires a precisely formed series of timing pulses. Typically, these may be of $5 \mu\text{s}$ duration with a repetition rate of 500 pulses per second. Starting with a 500-Hz sinusoidal generator, the pulse train could be developed as shown in Fig. 3.31. Can you visualize some relatively simple electronic circuits that would perform the indicated functions?

CLIPPING

To remove an undesired portion of a signal, we can use a *clipping circuit* consisting of a diode, a resistance, and a voltage source. In Fig. 3.32, the input signal v_1 varies with time as shown; we are interested in output signals v_R and v_D . The sum of the voltages around the loop is zero and the voltage relations are

$$v_D + v_R = v_1 - V \quad v_R = v_1 - V - v_D \quad v_D = v_1 - V - v_R \quad (3-32)$$

The behavior of the circuit depends on the state of the diode switch; it is closed when $v_D + v_R = v_1 - V$ (plotted in Fig. 3.32b) is positive. With the diode switch closed, $v_D = 0$ and $v_1 - V$ appears across R . At all other times, v_D is negative and the diode is open; no current flow in R and $v_R = 0$; this part of the signal has been *clipped*. In other words, the battery shifts the signal down and the diode cuts it off.

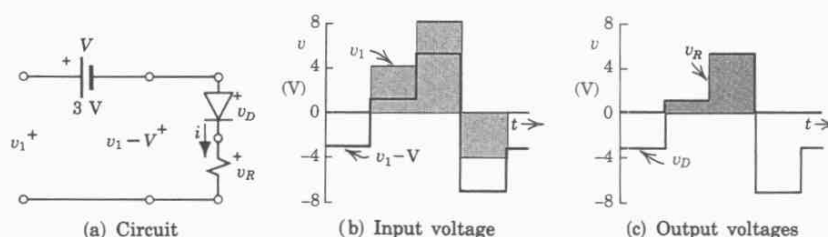


Figure 3.32 A clipping circuit.

The waveshaping function performed by the circuit depends on how we arrange the circuit components.

Exercise 3-12

- In the circuit of Fig. 3.32a, $v_1 = 6 \sin \omega t$ V. Sketch v_R for one cycle.
- Reverse the diode connections and repeat part (a).

Answers: (a) The signal below $v_1 = +3$ V is clipped; (b) $v_R = 0$ for $30^\circ < \omega t < 150^\circ$, $v_R = 6 \sin \omega t - 3$ V elsewhere.

One common type of *clipping circuit* provides an output voltage v_2 equal to (or proportional to) the input voltage v_1 up to a certain value V ; above V the wave

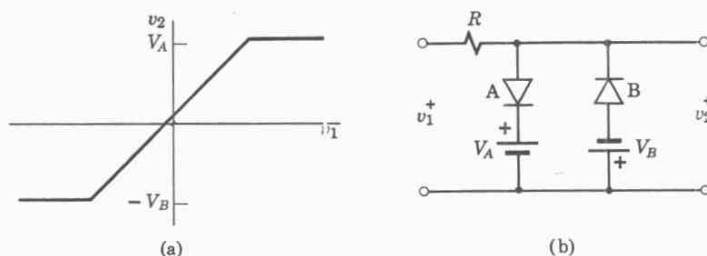


Figure 3.33 Clipping characteristic and circuit.

is clipped off. If both positive and negative peaks are to be clipped, the desired *transfer characteristic* v_2 versus v_1 is as shown in Fig. 3.33a. The diode circuit for this type of clipping is shown in Fig. 3.33b. The *bias* voltages are set so that diode A conducts whenever $v_1 > V_A$ and diode B conducts whenever $v_1 < -V_B$. When $-V_B < v_1 < V_A$, neither diode conducts and voltage v_1 appears across the output terminals. When either diode is conducting, the difference between v_1 and v_2 appears as a voltage drop across R . The effect of an asymmetric clipping circuit on a sinewave is illustrated in Example 11.

Example 11

A sinewave $v_1 = 20 \sin \omega t$ V is applied to the circuit of Fig. 3.34a. Draw the transfer characteristic and predict the output voltage v_2 .

In this circuit the first diode and the 10-V battery provide clipping for voltages greater than +10 V. With $V_B = 0$, the second diode provides clipping of all negative voltages or rectification. The transfer characteristic and the resulting output are as shown in Fig. 3.34b.

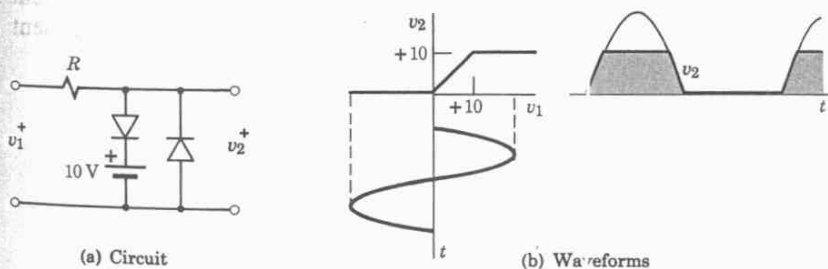


Figure 3.34 A clipping and rectifying circuit.

CLAMPING

To provide satisfactory pictures in television receivers, the peak values of certain variable signal voltages must be held or *clamped* at predetermined levels. In passing through ordinary amplifiers, the dc reference level is lost and a *clammer* or *dc restorer* is necessary to return the signal to its original form.

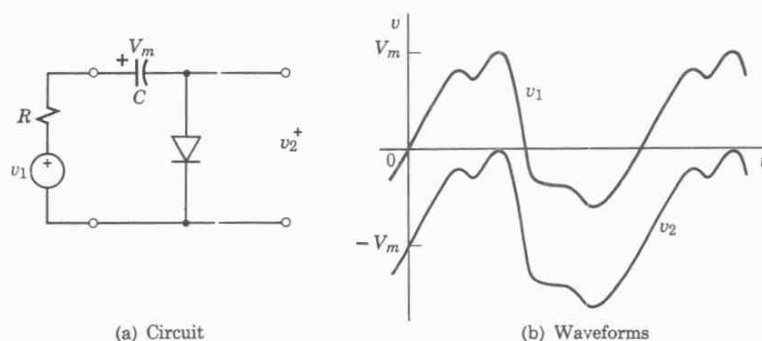


Figure 3.35 A diode clamping circuit.

In the circuit of Fig. 3.35a, if R is small the capacitor tends to charge up to the positive peak value of the input wave, just as in the half-wave rectifier with capacitor. When the polarity of v_1 reverses, the capacitor voltage remains at V_m because the diode prevents current flow in the opposite direction. Neglecting the small voltage across R , the output voltage across the diode is

$$v_2 = v_1 - V_m \quad (3-33)$$

The signal waveform is unaffected, but a dc value just equal to the peak value of the signal has been introduced. The positive peak is said to be clamped at zero.

If the amplitude of the input signal changes, the dc voltage across C also changes (after a few cycles) and the output voltage again just touches the axis. If the diode is reversed, the negative peaks are clamped at zero. If a battery is inserted in series with the diode, the reference level of the output may be maintained at voltage V_B .

Clamping and rectifying are related waveshaping functions performed by the same combination of diode and capacitor. In the rectifier the variable component is rejected and the dc value is transmitted; in the clamper the variable component is transmitted and the dc component is restored.

Example 12

Design a circuit that will clamp the minimum point of any periodic signal to -5 V.

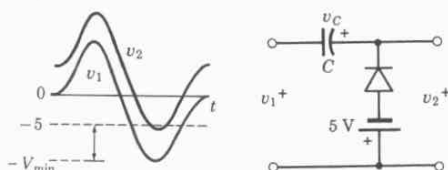


Figure 3.36 Designing a clamping circuit.

In Fig. 3.36, the output is to be

$$v_2 = v_1 + (V_{\min} - 5) = v_1 + v_C$$

Therefore, the capacitor must charge up to the voltage $v_C = V_{\min} - 5$ with the polarity shown.

When the input signal is negative with a magnitude greater than 5 V, current must flow through the diode, which must be connected as shown.

Exercise 3-13

The signal v_1 in Fig. 3.35 has a positive maximum $V_m = +12$ V and a negative maximum of -8 V.

- To what voltage v_C does the capacitor charge up?
- After C is charged, what is the value of v_2 when $v_1 = +6$ V?
- Reverse the diode connections in Fig. 3.35a and repeat parts (a) and (b).

Answers: (a) 12 V; (b) -6 V; (c) -8 V, $+14$ V.

DIFFERENTIATING

Within limits the simple circuit of Fig. 3.37 provides an output that is the derivative of the input. For the special case of a rectangular input voltage wave, the output voltage is proportional to the capacitor charging current in response to a step input voltage. In this case a linear circuit transforms a rectangular wave into a series of short pulses if the time constant RC is small compared to the period of the input wave.

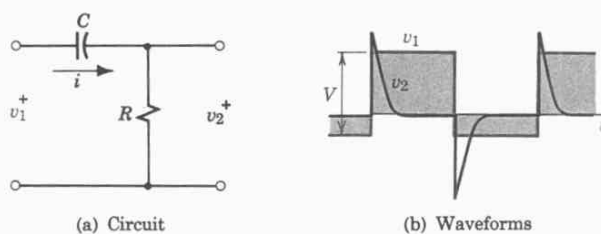


Figure 3.37 Differentiating circuit.

The general operation of this circuit is revealed if we make some simplifying assumptions. Applying Kirchhoff's voltage law to the left-hand loop,

$$v_1 = v_C + v_R \cong v_C \quad (3-34)$$

if v_R is small compared to v_C . Then

$$v_2 = v_R = Ri = RC \frac{dv_C}{dt} \cong RC \frac{dv_1}{dt} \quad (3-35)$$

The output is approximately proportional to the derivative of the input.

INTEGRATING

From our previous experience with circuits we expect that if differentiating is possible, integrating is also. In Fig. 3.38, a square wave of voltage has been applied long enough for a cyclic operation to be established. The time constant RC is a little greater than the half-period of the square wave. The capacitor C charges and discharges on alternate half-cycles, and the output voltage is as shown.

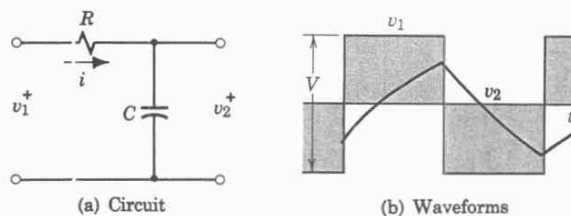


Figure 3.38 Integrating circuit.

If the time constant RC is large compared to the period T of the square waves, only the straight portion of the exponential appears and the output is the *sawtooth* wave in which voltage is directly proportional to time. In general,

$$v_1 = v_R + v_C \cong v_R = iR \quad (3-36)$$

if v_C is small compared to v_R (i.e., $RC > T$). Then

$$v_2 = \frac{1}{C} \int i \, dt \cong \frac{1}{RC} \int v_1 \, dt \quad (3-37)$$

and the output is approximately proportional to the integral of the input. If it is necessary, the magnitude of the signal can be restored by linear amplification.

OP AMP INTEGRATOR AND DIFFERENTIATOR

In the previous discussion of op amps we assumed that the feedback network is purely resistive. In general, however, the network may contain capacitances, inductances, and resistances. Because of the high amplifier gain in combination with feedback, the mathematical operations performed are precise.

In Fig. 3.39a, the feedback element is a capacitor. For the op amp, $v_i \cong 0$, node n is at ground potential, $i_i \cong 0$, and the sum of the currents into node n is

$$\frac{v_1}{R} + C \frac{dv_o}{dt} = 0 \quad \text{or} \quad dv_o = -\frac{v_1}{R_1 C} dt \quad (3-38)$$

Integrating each term with respect to time and solving,

$$v_o = -\frac{1}{R_1 C} \int v_1 \, dt + \text{a constant} \quad (3-39)$$

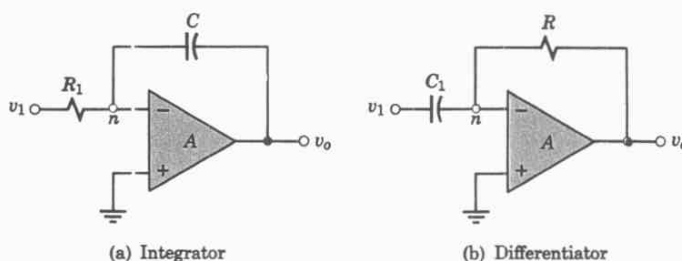


Figure 3.39 Op amp integration and differentiation circuits.

and the device is an *integrator*. The analog integrator is very useful in computing, signal processing, and signal generating

Example 13

Predict the output voltage of the circuit shown in Fig. 3.40 where the block represents an ideal amplifier with A very large.

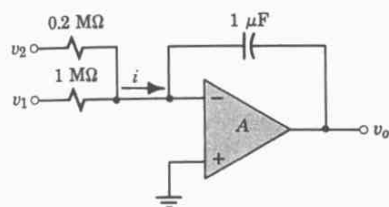


Figure 3.40 Integrator application.

For A very large, v_i is very small and representation as an integrator is accurate. Here $i = i_1 + i_2$ and

$$\begin{aligned} v_o &= -\frac{1}{C} \int \left(\frac{v_1}{R_1} + \frac{v_2}{R_2} \right) dt \\ &= -\frac{1}{CR_1} \int \left(v_1 + v_2 \frac{R_1}{R_2} \right) dt \\ &= -\int (v_1 + 5v_2) dt \end{aligned}$$

The output is the integral of a weighted sum.

If the resistance and capacitance are interchanged as in Fig. 3.39b, the sum of the currents is

$$C_1 \frac{dv_1}{dt} + \frac{v_o}{R} = 0 \quad (3-40)$$

Solving,

$$v_o = -RC_1 \frac{dv_1}{dt} \quad (3-41)$$

and the output voltage is proportional to the derivative of the input. For practical reasons involving instability and susceptibility to noise, the differentiator is not so useful as the integrator.

Exercise 3-14

For Example 13, let $v_1 = e^{-2t}$ and $v_2 = e^{-100t}$ for $t \geq 0$ and $v_o = 0$ at $t = 0$. Find and sketch v_o over the period $0 < t < 2.5$ s.

Answer: $v_o = 0.5 e^{-2t} + 0.05 e^{-100t} - 0.55$, $t \geq 0$.

SUMMARY

- Exponentials and sinusoids are important waveforms because: they occur frequently, they are easy to handle mathematically, and they are used to represent other waves.

The general decaying exponential is $a = A e^{-t/\tau}$.

The time constant τ is a measure of the rate of decay.

For $t = \tau$, $a/A = 1/e = 0.368$; for $t = 5\tau$, $a/A = 0.0067$ (negligible).

- The general sinusoid is $a = A \cos(\omega t + \alpha)$.
Frequency $\omega = 2\pi f$ rad/s; $f = 1/T$ Hz, where period T is in seconds.
- In a periodic function of time, $f(t + T) = f(t)$.
The average value of a periodic current is $I_{av} = (1/T) \int_0^T i \, dt$.
For a sinusoid, the half-cycle average is $2I_m/\pi = 0.637I_m$.
The effective or rms value of a periodic current is $I = \sqrt{(1/T) \int_0^T i^2 \, dt}$.
For a sinusoid, the effective value is $I = I_m/\sqrt{2} = 0.707I_m$.
- An ideal amplifier is characterized by: infinite input resistance, zero output resistance, and constant gain; an ideal op amp has infinite gain.
Feedback circuits are used with op amps to obtain: inverting and noninverting amplifiers, summing circuits, integrators, and differentiators.
- Essentially, a diode discriminates between forward and reverse voltages.
The ideal diode presents zero resistance in the forward direction and infinite resistance in the reverse direction; it functions as a selective switch.
- A rectifier converts alternating current into unidirectional current.
In half-wave rectification with a resistive load, $I_{dc} \cong V_m/\pi R_L = I_m/\pi$.
A bridge circuit or phase inverter permits full-wave rectification; $I_{dc} = 2I_m/\pi$.
- A capacitor filter stores charge on voltage peaks and delivers charge during voltage valleys; the ripple voltage (half-wave) is $V_r \cong V_{dc}/fCR_L$.
- Waveforms can be shaped easily, rapidly, and precisely.
A diode-resistor-battery circuit can perform clipping.
A diode and peak-charging capacitor can clamp signals to desired levels.
An op amp can perform differentiation and integration precisely.

TERMS AND CONCEPTS

diode Two-terminal device that acts as a switch; it permits current to flow readily in one direction but tends to prevent the flow of current in the other direction.

effective value of a current Steady current that is as effective in transferring power as the given varying current.

filter Circuit passing signals of selected frequencies while rejecting signals of different frequencies.

operational amplifier Amplifier with a high gain designed to be used with other circuit components to perform a specified signal-processing function.

rectifier Device for changing alternating current to unidirectional current.

signal, electric Voltage or current varying with time in a manner that conveys information.

sinusoidal waveform Voltage or current variation in accordance with a sine or cosine function of time.

time constant Measure of the rate of decay of an exponential function; equal to the time for an exponential response to decrease to 37% of its initial value.

waveform Pattern of time variation of a voltage or current.