Math 15 Name ________________________________
Sections 1 & 2
Fall 2009

Exam I

INSTRUCTIONS: This exam is seven pages long; check that you have all the pages. Be sure to show all your work on the attached sheets. No credit will be given for unsubstantiated solutions. Also, where appropriate, please indicate your final answers by circling them.

You may use a calculator, but **for arithmetic only**—no graphing, equation solving, or calculus-based features. Mark the places where you've used a calculator by writing a "C" in the left margin. Feel free to ask during the exam if any questions arise about calculator use (or about anything else for that matter).

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1. A particle moves along a straight line in such a way that its position as a function of time is given by the following table. (12 points)

Time (sec)	Position (ft)
0	0
0.1	1.80
0.2	3.25
0.3	4.50
0.4	5.60
0.5	6.60

(a) Let $f(t)$ denote the object's position at time t. Estimate the value of $f'(t)$ at each of the times in the table below. Give the best estimates that the data allows. If you do not show the details of your calculations, please write a brief description of what you did to the right of the table. What are the units of your answers?

t	$f'(t)$
0	
0.1	
0.2	
0.3	
0.4	
0.5	

(b) Based on the information that you've collected, is there any time at which the second derivative $f''(t)$ is positive? If so, identify such a time and estimate the value of $f''(t)$ at that instant. If no such time exists, explain why not.

2. Find the derivatives of the following functions. Simplify your answers to a reasonable degree. (15 points)

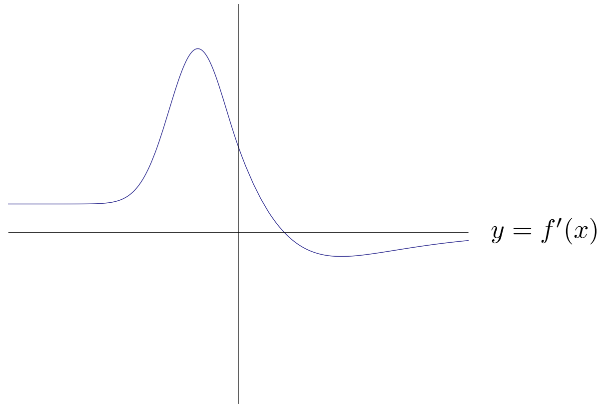
(a) $f(x) = x^3 - 4x^2 + 7x + 3$

(b) $f(x) = e^{-2x} \sin x$

(c) $f(x) = (\ln(x^2 + 1))^2$

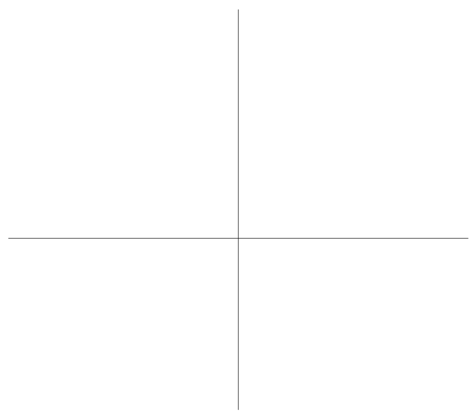
3. The curve below is the graph of the *derivative*, $y = f'(x)$, of a certain function.

(15 points)

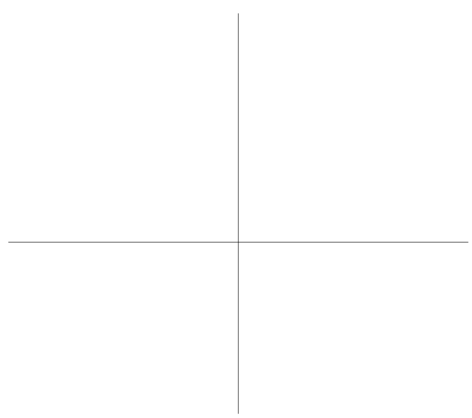


In the following problems, you are asked to sketch the graphs of the second derivative f'' and the original function f. You need not construct graphs that are completely accurate, but incorporate as much of the available information as you can.

(a) Sketch the graph of the second derivative f''.



(b) Sketch a possible graph of the original function f.



4. **(a)** State the product rule for the derivative of a product of functions. (12 points)

(b) Give a proof of the product rule starting from the definition of the derivative as a limit of difference quotients.

5. Your enthusiasm E for calculus after t weeks of this course is described by the function:

$$E(t) = 2^{t^2} \text{ units of enthusiasm, where } t \geq 0.$$

(16 points)

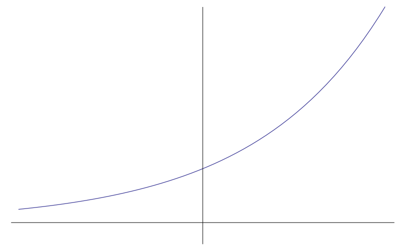
(a) What was the average rate of change of your enthusiasm over the first two weeks of the course, that is, over the interval $0 \leq t \leq 2$?

(b) Find $E'(t)$.

(c) Find $E(2)$ and $E'(2)$. Then interpret what your answers mean in everyday language. Be clear and concise, yet complete.

(d) Find $E^{-1}(2)$ and $(E^{-1})'(2)$, and give clear and thorough interpretations of what your answers mean. As usual, be sure to show enough work to justify your solution.

6. Find an equation of the line that passes through the origin and is tangent to the curve $y = e^{7x}$. (10 points)



7. The following equation has a solution near $x = 0$:

$$\sin 3x - \ln(1 + x) = 0.1.$$

By replacing the left side of the equation with its tangent line approximation at $x = 0$, find an approximate value for the solution. Show all steps. (10 points)

1.	
2.	
3.	
4.	
5.	
6.	
7.	
Total	**/90**