FUTHER TOPICS: SINGLE VARIABLE CALUCULUS

January 20, 2012

Group Work I

Your Group Number: .....

Your Names in alphabetical order **(Print last name first)**:

1. KEY .....

2. .....

3. .....

4. .....

Instructions

There are 3 questions. Each question is worth 5 points.
Write your answer in the space provided after each question.
Show your work to get full credit (unless instructed otherwise) - No credit is given for unsubstantiated answers!
Open Notes - Calculators are allowed!

Question 1

The **error function**

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

is used in probability, statistics, and engineering.

(a) Show that $\displaystyle\int_a^b e^{-t^2} dt = \frac{1}{2}\sqrt{\pi}\,[\text{erf(b)} - \text{erf}(a)]$

$$\begin{aligned}\int_a^b e^{-t^2} dt &= \int_a^0 e^{-t^2} dt + \int_0^b e^{-t^2} dt \\ &= \int_0^b e^{-t^2} dt - \int_0^a e^{-t^2} dt \\ &= \frac{1}{2}\sqrt{\pi}\,[\text{erf(b)} - \text{erf}(a)]\end{aligned}$$

(b) Show that the function $y = e^{x^2}\text{erf}(x)$ satisfies the differential equation $y' = 2xy + 2/\sqrt{\pi}$.

$$\begin{aligned}y' &= (e^{x^2})'\text{erf}(x) + e^{x^2}(\text{erf}(x))', \quad \text{by the product rule} \\ &= 2xe^{x^2}\text{erf}(x) + e^{x^2}\left(\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt\right) \\ &= 2xe^{x^2}\text{erf}(x) + e^{x^2}\frac{2}{\sqrt{\pi}}e^{-x^2} \\ &= 2xy + \frac{2}{\sqrt{\pi}}\end{aligned}$$

Question 2

A helicopter on the ground is rising straight up in the air. Its velocity at time t is $2t + 1$ feet per second.

(a) How high does the helicopter rise during the first 5 seconds?

$$\begin{aligned}\int_0^5 (2t+1)dt &= (t^2 + t)|_0^5 \\ &= 30 \;\; feet\end{aligned}$$

(b) Sketch the graph of the velocity function and then represent the answer to part (a).

Question 3

Evaluate the integral

$$\int_0^{\pi/3} \frac{\sin(\theta) + \sin(\theta)\tan^2(\theta)}{\sec^2(\theta)}\, d\theta.$$

$$\begin{aligned}\int_0^{\pi/3} \frac{\sin(\theta) + \sin(\theta)\tan^2(\theta)}{\sec^2(\theta)}\, d\theta &= \int_0^{\pi/3} \frac{\sin(\theta)(1+\tan^2(\theta))}{\sec^2(\theta)}\, d\theta \\ &= \int_0^{\pi/3} \frac{\sin(\theta)\sec^2(\theta)}{\sec^2(\theta)}\, d\theta \\ &= \int_0^{\pi/3} \sin\theta\, d\theta = -(\cos(\pi/3) - \cos 0) = \frac{1}{2}\end{aligned}$$