SECTION 21.1 Introduction

In earlier chapters, we dealt with techniques for manipulating data in order to make decisions about population parameters and population characteristics. Our focus in this chapter is also on decision making, but the types of problems we deal with here differ in several ways. First, the technique for hypothesis testing concludes with either rejecting or not rejecting some hypothesis concerning a dimension of a population. In decision analysis, we deal with the problem of selecting one alternative from a list of several possible decisions. Second, in hypothesis testing the decision is based on the statistical evidence available. In decision analysis, there may be no statistical data, or if there are data, the decision may depend only partly on them. Third, costs (and profits) are only indirectly considered (in the selection of a significance level) in the formulation of a hypothesis test. Decision analysis directly involves profits and losses. Because of these major differences, the only topics covered previously in the text that are required for an understanding of decision analysis are Bayes' theorem and calculations of expected value.

SECTION 21.2 Decision Problem

You would think that, by this point in the text, we would already have introduced all the necessary concepts and terminology. Unfortunately, because decision analysis is so radically different from statistical inference, several more terms need to be defined. They will be introduced in the following example.

EXAMPLE 21.1

A man wants to invest $1 million for one year. After analyzing and eliminating numerous possibilities, he has narrowed his choice to one of three alternatives. These alternatives are referred to as **acts** and are denoted a_i:

a_1: Invest in a guaranteed income certificate paying 10%.

a_2: Invest in a bond with a coupon value of 8%.

a_3: Invest in a portfolio of banking institution stocks.

He believes that the payoffs associated with the last two acts depend on a number of factors, foremost among which is interest rates. He concludes that there are three possible **states of nature,** denoted s_j:

s_1: Interest rates increase.

s_2: Interest rates stay the same.

s_3: Interest rates decrease.

After further analysis, he determines the amount of profit he will make for each possible combination of one act and one state of nature. Of course, the payoff for the guaranteed income certificate will be $100,000 no matter which state of nature occurs. The profits from each alternative investment are summarized in Table 21.1, i what is called a **payoff table.** Notice that, when the decision is a_2 and the state of nature is s_1, the investor would suffer a \$50,000 loss, which is represented by a −\$50,000 payoff.

Another way of expressing the consequence of an act involves measuring the opportunity loss associated with each combination of one act and one state of nature. An **opportunity loss** is the difference between what the decision maker's profit for an act is and what the profit could have been had the best decision been made. For example, consider the first row of Table 21.1. If s_1 is the state of nature that occurs and the investor chooses act a_1, he makes a profit of \$100,000. However, had he chosen act a_3, he would have made a profit of \$150,000. The difference between what he could have made (\$150,000) and what he actually made (\$100,000) is the opportunity loss. Thus, given that s_1 is the state of nature, the opportunity loss of act a_1 is \$50,000. The opportunity loss of act a_2 is \$200,000, which is the difference between \$150,000 and −\$50,000. The opportunity loss of act a_3 is 0, since there is no opportunity loss when the best alternative is chosen. In a similar manner, we can compute the remaining opportunity losses for this example (see Table 21.2). Notice that we can never experience a negative opportunity loss.

Decision Trees

Most problems involving a simple choice of alternatives can readily be resolved by using the payoff table (or the opportunity loss table). In other situations, however, the decision maker must choose between sequences of acts. In Section 21.5, we introduce one form of such situations. In these cases, a payoff table will not suffice to determine the best alternative; instead, we require a **decision tree.**

In Chapter 4, we suggested the probability tree as a useful device for computing probabilities. In this type of tree, all the branches represent stages of events. In a

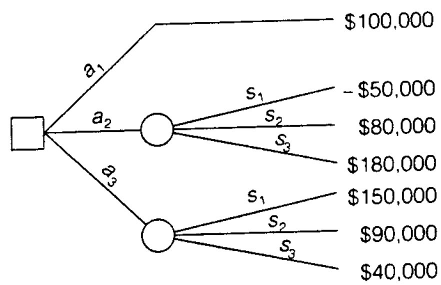
TABLE 21.1 Payoff Table for Example 21.1

	a_1 (GIC)	a_2 (bond)	a_3 (stocks)
s_1 (interest rates increase)	\$100,000	−\$ 50,000	\$150,000
s_2 (interest rates stay the same)	100,000	80,000	90,000
s_3 (interest rates decrease)	100,000	180,000	40,000

TABLE 21.2 Opportunity Loss Table for Example 21.1

	a_1	a_2	a_3
s_1	\$50,000	\$200,000	\$ 0
s_2	0	20,000	10,000
s_3	80,000	0	140,000

FIGURE 21.1 Decision Tree for Example 21.1



decision tree, however, the branches represent both acts and events (states of nature). We distinguish between them in the following way: a point where a decision is to be made is represented by a square node; a point where a state of nature occurs is represented by a round node. Figure 21.1 depicts the decision tree for Example 21.1.

The tree in Figure 21.1 begins with a square node; that is, we begin by making a choice among a_1, a_2, and a_3. The branches emanating from the square node represent these alternatives. At the ends of branches a_2 and a_3, we reach round nodes representing the occurrence of some state of nature. These are depicted as branches representing s_1, s_2, and s_3. At the end of branch a_1, we don't really have a state of nature, because the payoff is fixed at $100,000 no matter what happens to interest rates.

At the ends of the branches, the payoffs are shown (alternatively, we could have worked with opportunity losses instead of with payoffs). These are, of course, the same values that appear in Table 21.1.

Up to this point, all we have done is set up the problem; we have not made any attempt to determine the decision. It should be noted that in many real-life problems determining the payoff table or decision tree can be a formidable task in itself. Many managers, however, have observed that this task is often extremely helpful in decision making.

EXERCISES

Learning the Techniques

21.1 Set up the opportunity loss table from the following payoff table.

	a_1	a_2
s_1	55	26
s_2	43	38
s_3	29	43
s_4	15	51

21.2 Given the following payoff table, draw the decision tree.

	a_1	a_2	a_3
s_1	20	5	−1
s_2	8	5	4
s_3	−10	5	10

21.3 Consider the following payoff table.

	a_1	a_2	a_3
s_1	−20	−30	−25
s_2	−10	−2	−5

a. Set up the opportunity loss table.
b. Draw the decision tree.

21.4 Set up the opportunity loss table from the following payoff table.

	a_1	a_2	a_3	a_4
s_1	200	100	110	80
s_2	75	120	110	90
s_3	50	150	110	200

21.5 Draw the decision tree for Exercise 21.4.

Applying the Techniques

Self-Correcting Exercise

(See Appendix 21.A for the solution.)

21.6 A baker must decide how many specialty cakes to bake each morning. From past experience, he knows that the daily demand for cakes ranges from 0 to 3. If each cake costs $3.00 to produce and sells for $8.00 and if any unsold cakes are thrown into the garbage at the end of the day, set up a payoff table to help the baker decide how many cakes to bake.

21.7 Set up the opportunity loss table for Exercise 21.6.

21.8 Draw the decision tree for Exercise 21.6.

21.9 The manager of a large shopping center in Buffalo is in the process of deciding on the type of snow-clearing service to hire for his parking lot. Two services are available. The White Christmas Company will clear all snowfalls for a flat fee of $70,000 for the entire winter season. The We-plowem Company charges $18,000 for each snowfall it clears. Set up the payoff table to help the manager decide, assuming that the number of snowfalls per winter season is Poisson-distributed with $\mu = 3.0$.

21.10 Draw the decision tree for Exercise 21.9.

21.11 The owner of a clothing store must decide how many men's shirts to order for the new season. For a particular type of shirt, she must order in quantities of 100 shirts. If she orders 100 shirts, her cost is $10 per shirt; if she orders 200 shirts, her cost is $9 per shirt; and if she orders 300 or more shirts, her cost is $8.50 per shirt. Her selling price for the shirt is $12, but any shirts that remain unsold at the end of the season are sold at her famous "half-price, end-of-season sale." For the sake of simplicity, she is willing to assume that the demand for this type of shirt will be either 100, 150, 200, or 250 shirts. Of course, she cannot sell more shirts than she stocks. She is also willing to assume that she will suffer no loss of goodwill among her customers if she understocks and the customers cannot buy all the shirts they want. Furthermore, she must place her order today for the entire season; she cannot wait to see how the demand is running for this type of shirt. Construct the payoff table to help the owner decide how many shirts to order.

21.12 Set up the opportunity loss table for Exercise 21.11.

21.13 Draw the decision tree for Exercise 21.11.

21.14 A building contractor must decide how many mountain cabins to build in the ski resort area of Chick-oh-pee. He builds each cabin at a cost of $26,000 and sells each for $33,000. All cabins unsold after 10 months will be sold to a local investor for $20,000. The contractor believes that the demand for cabins follows a Poisson distribution, with a mean of .5. He assumes that any probability less than .01 can be treated as zero. Construct the payoff table and the opportunity loss table for this decision problem.

SECTION 21.3 Expected Monetary Value Decisions

In many decision problems, it is possible to assign probabilities to the states of nature. For example, if the decision involves trying to decide whether or not to draw to an inside straight in the game of poker, the probability of succeeding can easily be determined by the use of simple rules of probability. If we must decide whether or not to replace a machine that has broken down frequently in the past, we can assign probabilities on the basis of the relative frequency of the breakdowns. In many other instances, however, formal rules and techniques of probability cannot be applied. In

Example 21.1, the historical relative frequencies of the ups and downs of interest rates will supply scant useful information to help the investor assign probabilities to the behavior of interest rates during the coming year. In such cases, probabilities must be assigned subjectively. In other words, the determination of the probabilities must be based on the experience, knowledge, and (perhaps) guesswork of the decision maker.

If in Example 21.1 the investor has some knowledge about a number of economic variables, he might have a reasonable guess about what will happen to interest rates in the next year. Suppose, for example, that our investor believes that future interest rates are most likely to remain essentially the same as they are today and that (of the remaining two states of nature) rates are more likely to decrease than to increase. He might then guess the following probabilities:

$$P(s_1) = .2,\ P(s_2) = .5,\ P(s_3) = .3$$

Because the probabilities are subjective, we would expect another decision maker to produce a completely different set of probabilities. In fact, if this were not true, we would rarely have buyers and sellers of stocks (or any other investment), because everyone would be a buyer (and there would be no sellers) or everyone would be a seller (with no buyers).

After determining the probabilities of the states of nature, we can address the expected monetary value (EMV) decision.

Expected Monetary Value Decision

We now calculate what we expect will happen for each decision. Because we generally measure the consequences of each decision in monetary terms, we compute the **expected monetary value (EMV)** of each. Recall from Chapter 5 that we calculate expected values by multiplying the values of the random variables by their respective probabilities and then summing the products. Thus, in our example, the expected monetary value of alternative a_1 is

$$\text{EMV}(a_1) = .2(100{,}000) + .5(100{,}000) + .3(100{,}000) = \$100{,}000$$

The expected values of the other decisions are found in the same way:

$$\text{EMV}(a_2) = .2(-50{,}000) + .5(80{,}000) + .3(180{,}000) = \$84{,}000$$

$$\text{EMV}(a_3) = .2(150{,}000) + .5(90{,}000) + .3(40{,}000) = \$87{,}000$$

We choose the decision with the largest expected monetary value, which is a_1, and label its expected value EMV*. Hence, EMV* = $100,000.

In general, the expected monetary values do not represent possible payoffs. For example, the expected monetary value of act a_2 is $84,000, yet the payoff table indicates that the only possible payoffs from choosing a_2 are −$50,000, $80,000, or $180,000. Of course, the expected monetary value of act a_1 ($100,000) is possible, because that is the only payoff of the act.

What, then, does the expected monetary value represent? If the investment is made a large number of times, with exactly the same payoffs and probabilities, the expected monetary value is the average payoff per investment. That is, if the invest ment is repeated an infinite number of times with act a_2, 20% of the investments will result in a $50,000 loss, 50% will result in an $80,000 profit, and 30% will result in a $180,000 profit. The average of all these investments is the expected monetary value, $84,000. If act a_3 is chosen, the average payoff in the long run will be $87,000.

An important point is raised by the question of how many investments are going to be made. The answer is one. Even if the investor intends to make the same type of investment annually, the payoffs and the probabilities of the states of nature will undoubtedly change from year to year. Hence, we are faced with having computed the expected monetary value decision on the basis of an infinite number of investments, when there will actually be only one investment. We can rationalize this apparent contradiction in two ways. First, the expected value decision is the only method that allows us to combine the two most important factors in the decision process—the payoffs and their probabilities. It seems inconceivable that, where both factors are known, the investor would want to ignore either one. (There are processes that make decisions on the basis of the payoffs alone; however, these processes assume no knowledge of the probabilities, which is not the case with our example.) Second, typical decision makers make a large number of decisions over their lifetimes. By using the expected value decision, the decision maker should perform at least as well as anyone else. Thus, despite the problem of interpretation, we advocate the expected value decision.

Expected Opportunity Loss Decision

We can also calculate the **expected opportunity loss (EOL)** of each act. From the opportunity loss table (Table 21.2), we get the following values:

$$\text{EOL}(a_1) = .2(50{,}000) + .5(0) + .3(80{,}000) = \$34{,}000$$

$$\text{EOL}(a_2) = .2(200{,}000) + .5(20{,}000) + .3(0) = \$50{,}000$$

$$\text{EOL}(a_3) = .2(0) + .5(10{,}000) + .3(140{,}000) = \$47{,}000$$

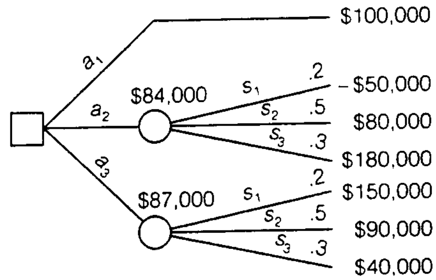
Because we want to minimize losses, we choose the act that produces the smallest expected opportunity loss, which is a_1. We label its expected value EOL*.

Observe that the EMV decision is the same as the EOL decision. This is not a coincidence—the opportunity loss table was produced directly from the payoff table.

Rollback Technique for Decision Trees

Figure 21.2 presents the decision tree for Example 21.1, with the probabilities of the states of nature included. The process of determining the EMV decision is called the *rollback technique;* it operates as follows. Beginning at the end of the tree (right-hand side), we calculate the expected monetary value at each round node. The numbers above the round nodes in Figure 21.2 specify these expected monetary values. At each square node, we make a decision by choosing the branch with the largest EMV. In our example, there is only one square node. Our optimal decision is, of course, a_1.

FIGURE 21.2 Rollback Technique for Example 21.1



EXERCISES

Learning the Techniques

21.15 Given the following payoff table and probabilities, determine the expected monetary values and the optimal act.

	a_1	a_2	a_3
s_1	10	5	13
s_2	20	25	13
s_3	15	12	15

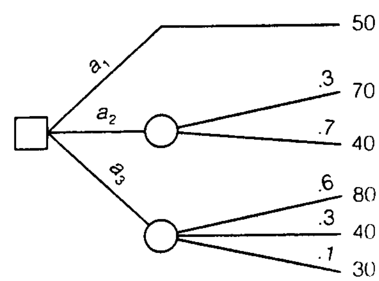
$P(s_1) = .45, P(s_2) = .25, P(s_3) = .30$

21.16 Find the opportunity loss table from the following payoff table. Given these results and the accompanying probabilities, compute the expected opportunity loss for each act, and specify the optimal act.

	a_1	a_2	a_3	a_4
s_1	25	20	35	5
s_2	50	60	40	100

$P(s_1) = .7, P(s_2) = .3$

21.17 Use the rollback technique on the accompanying decision tree to determine the optimal act.



21.18 For the accompanying payoff table and probabilities, draw the decision tree; then use the rollback technique to determine the optimal act.

	a_1	a_2
s_1	−5	−12
s_2	3	−5
s_3	17	25
s_4	28	50

$P(s_1) = .10, P(s_2) = .20, P(s_3) = .45, P(s_4)$

21.19 Refer to the accompanying payoff table and probabilities.

	a_1	a_2	a_3
s_1	15	0	12
s_2	18	17	20
s_3	22	53	33

$P(s_1) = .6, P(s_2) = .3, P(s_3) = .1$

a. Calculate the optimal act, and determine EMV*.

b. Set up the opportunity loss table, and find EOL*.

Applying the Techniques

Self-Correcting Exercise

(See Appendix 21.A for the solution.)

21.20 The electric company is in the process of building a new power plant. There is some uncertainty regarding the size of the plant to be built. If the community that the plant will service attracts a large number of industries, the demand for electricity will be high. If commercial establishments (offices and retail stores) are attracted, demand will be moderate. If neither industries nor commercial stores locate in the community, the electricity demand will be low. The company can build a small, medium, or large plant, but if the plant is too small, the company will incur extra costs. The total costs (in $millions) of all options are shown in the accompanying table.

DEMAND FOR ELECTRICITY	SIZE OF PLANT		
	Small	Medium	Large
Low	220	300	350
Moderate	330	320	350
High	440	390	350

The following probabilities are assigned to the electricity demand.

DEMAND	P(DEMAND)
Low	.15
Moderate	.55
High	.30

a. Determine the act with the largest expected monetary value. (CAUTION: All the values in the table are costs.)

b. Draw up an opportunity loss table.

c. Calculate the expected opportunity loss for each decision, and determine the optimal decision.

21.21 A retailer buys bushels of mushrooms for $2 each and sells them for $5 each. The quality of the mushrooms begins to decline after the first day they are offered for sale; therefore, in order to sell the mushrooms for $5/bushel, he must sell them on the first day. Bushels not sold on the first day can be sold to a wholesaler who buys day-old mushrooms at the following rate.

AMOUNT PURCHASED (bushels)	1	2	3	4 OR MORE
Price per bushel	$2.00	$1.75	$1.50	$1.25

A 90-day observation of past demand yields the following information.

Daily demand (bushels)	10	11	12	13
Number of days	9	18	36	27

a. Set up a payoff table that could be used by the retailer to decide how many bushels to buy.

b. Find the optimal number of bushels the retailer should buy in order to maximize profit.

21.22 For Exercise 21.9, determine the act with the largest expected monetary value.

21.23 An international manufacturer of electronic products is contemplating introducing a new type of compact disk player. After some analysis of the market, the president of the company concludes that, within two years, the new product will have a market share of 5%, 10%, or 15%. He assesses the probabilities of these events as .15, .45, and .40, respectively. The vice-president of finance informs him that, if the product captures only a 5% market share, the company will lose $28 million. A 10% market share will produce a $2-million profit, and a 15% market share will produce an $8-million profit. If the company decides not to begin production of the new compact disk player, there will be no profit or loss. Based on the expected value decision, what should the company do?

21.24 A retailer of microcomputers has an opportunity to buy a lot of 20 semiobsolete computers. She can purchase the entire lot for $30,000. The selling price she will be able to ask depends on whether a new line of computers is forthcoming in the next three months. If the new line is brought out, the 20 computers will be fully obsolete, and she will be able to sell them for only $1,000 each. On the other hand, if the new line is delayed, she will be able to sell her computers for $2,200 each. If the probability that the new line will be delayed is .40, should she purchase the lot?

SECTION 21.4 Expected Value of Perfect Information

In Section 21.5, we discuss methods of introducing and incorporating additional information into the decision process. Such information generally has value, but it also has attendant costs; that is, we can acquire useful information from consultants, surveys, or other experiments, but we usually must pay for this information. In this section, we calculate the maximum price that a decision maker should be willing to pay for any information by determining the value of perfect information. We begin by calculating the **expected payoff with perfect information (EPPI).**

If we knew in advance which state of nature would occur, we would certainly make our decisions accordingly. For instance, if the investor in Example 21.1 knew before investing his money what interest rates would do, he would choose the best act to suit that case. Referring to Table 21.1, if he knew that s_1 was going to occur, he would choose act a_3; if s_2 were certain to occur, he'd choose a_1, and if s_3 were certain, he'd choose a_2. Thus, in the long run, his expected payoff from perfect information would be

$$\text{EPPI} = .2(150{,}000) + .5(100{,}000) + .3(180{,}000) = \$134{,}000$$

Notice that we compute EPPI by multiplying the probability of each state of nature by the largest payoff associated with that state of nature and then summing the products.

This figure, however, does not represent the maximum amount he'd be willing to pay for perfect information. Because the investor could make an expected profit of EMV* = $100,000 without perfect information, we subtract EMV* from EPPI to determine the **expected value of perfect information (EVPI).** That is,

$$\text{EVPI} = \text{EPPI} - \text{EMV*} = \$134{,}000 - \$100{,}000 = \$34{,}000$$

This means that, if perfect information were available, the investor should be willing to pay up to $34,000 to acquire it.

You may have noticed that the expected value of perfect information (EVPI) equals the smallest expected opportunity loss (EOL*). Again, this is not a coincidence—it will always be the case. In future questions, if the opportunity loss table has been determined, you need only calculate EOL* in order to know EVPI.

EXERCISES

Learning the Techniques

21.25 Find EPPI, EMV*, and EVPI for the accompanying payoff table and probabilities.

	a_1	a_2	a_3
s_1	60	110	75
s_2	40	110	150
s_3	220	120	85
s_4	250	120	130

$P(s_1) = .10$, $P(s_2) = .25$, $P(s_3) = .50$, $P(s_4)$ ...

21.26 For Exercise 21.25, determine the opportunity loss table and compute EOL*. Confirm that EOL* = EVPI.

21.27 Given the following payoff table and probabilities, determine EVPI.

	a_1	a_2	a_3	a_4
s_1	65	20	45	30
s_2	70	110	80	95

$P(s_1) = .5, P(s_2) = .5$

21.28 Redo Exercise 21.27, changing the probabilities to the following values.

a. $P(s_1) = .75, P(s_2) = .25$

b. $P(s_1) = .95, P(s_2) = .05$

21.29 What conclusion can you draw about the effect of the probabilities on EVPI from Exercises 21.27 and 21.28?

Applying the Techniques

Self-Correcting Exercise

(See Appendix 21.A for the solution.)

21.30 A sporting-goods store owner has the opportunity to purchase a lot of 50,000 footballs for $100,000. She believes that she can sell some or all by taking out mail-order advertisements in a magazine. Each football will be sold for $6.00. The advertising cost is $25,000, and the mailing cost per football is $1.00. She believes that the demand distribution is as follows.

DEMAND	P(DEMAND)
10,000	.2
30,000	.5
50,000	.3

What is the maximum price the owner should pay for additional information about demand?

21.31 Calculate the expected value of perfect information for the decision problem described in Exercise 21.20.

21.32 What is the maximum price the electronics product manufacturer should be willing to pay for perfect information regarding the market share in Exercise 21.23?

21.33 A radio station currently directing its programming toward middle-age listeners is contemplating switching to rock-and-roll music. After analyzing advertising revenues and operating costs, the owner concludes that, for each percentage point of market share, revenues increase by $100,000 per year. Fixed annual operating costs are $700,000. The owner believes that, with the change, the station will get a 5%, 10%, or 20% market share, with probabilities .4, .4, and .2, respectively. The current annual profit is $285,000.

a. Set up the payoff table.

b. Determine the optimal act.

c. What is the most the owner should be willing to pay to acquire additional information about the market share?

21.34 What is the most the retailer in Exercise 21.24 should be willing to pay for any additional information?

SECTION 21.5 Using Additional Information

Suppose the investor in our continuing example wants to improve his decision-making capabilities. He learns about Investment Management Consultants (IMC), who, for a fee of $5,000, will analyze the economic conditions and forecast the behavior of interest rates over the next 12 months. The investor, who is quite shrewd (after all, he does have $1 million to invest), asks for some measure of IMC's past successes. IMC has been forecasting interest rates for many years and so provides him with various conditional probabilities (referred to as **likelihood probabilities**), as shown in Table 21.3.

Table 21.3 uses the following notation:

I_1: IMC predicts that interest rates will increase.

I_2: IMC predicts that interest rates will stay the same.

I_3: IMC predicts that interest rates will decrease.

TABLE 21.3 Likelihood Probabilities $P(I_i \mid s_j)$

	I_1 (predict s_1)	I_2 (predict s_2)	I_3 (predict s_3)
s_1	$P(I_1 \mid s_1) = .60$	$P(I_2 \mid s_1) = .30$	$P(I_3 \mid s_1) = .10$
s_2	$P(I_1 \mid s_2) = .10$	$P(I_2 \mid s_2) = .80$	$P(I_3 \mid s_2) = .10$
s_3	$P(I_1 \mid s_3) = .10$	$P(I_2 \mid s_3) = .20$	$P(I_3 \mid s_3) = .70$

The I_i terms are referred to as **experimental outcomes,** and the process by which we gather additional information is called the **experiment.**

Examine the first line of the table. When s_1 actually did occur in the past, IMC correctly predicted s_1 60% of the time; 30% of the time, it predicted s_2; and 10% of the time, it predicted s_3. The second row gives the conditional probabilities of I_1, I_2, and I_3 when s_2 actually occurred. The third row shows the conditional probabilities when s_3 actually occurred.

The following question now arises: How is the investor going to use the forecast that IMC produces? One approach is simply to assume that whatever it forecasts will actually take place and to choose the act accordingly. There are several drawbacks to this approach. Foremost among them is that it puts the investor in the position of ignoring whatever knowledge (in the form of subjective probabilities) he had concerning the issue. The optimal way to use the forecast is to incorporate the investor's subjective probabilities with the consultant's forecast. The medium for doing this is Bayes' theorem (developed in Chapter 4).

Suppose for now that the investor pays IMC the $5,000 fee; IMC does its work and finally forecasts that s_1 will occur. We want to determine the probabilities of the states of nature, given that I_1 is the outcome of the experiment. That is, what are $P(s_1 \mid I_1)$, $P(s_2 \mid I_1)$, and $P(s_3 \mid I_1)$? Before proceeding, let's develop some terminology.

The original subjective probabilities, $P(s_1)$, $P(s_2)$, and $P(s_3)$, are called **prior probabilities,** because they were determined prior to the acquisition of any additional information. In this example, they were based on the investor's experience. The set of probabilities we want to compute—$P(s_1 \mid I_1)$, $P(s_2 \mid I_1)$, and $P(s_3 \mid I_1)$—are called **posterior** or **revised probabilities.**

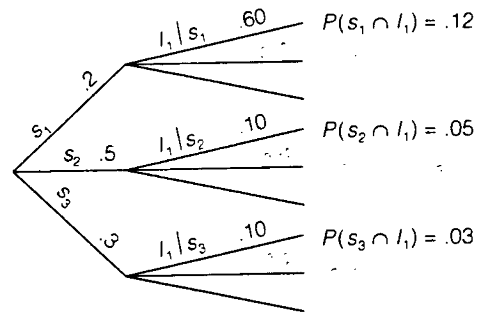
Now we will calculate the posterior probabilities, first by using a probability tree (as was done in Chapter 4) and then by applying a less time-consuming method. Figure 21.3 depicts the probability tree. We begin with the branches of the prior probabilities, which are followed by the likelihood probabilities.

Notice that we label only $P(I_1 \mid s_1)$, $P(I_1 \mid s_2)$, and $P(I_1 \mid s_3)$, because (at this point) we are assuming that I_1 is the experimental outcome. Thus,

$$P(s_1 \mid I_1) = \frac{P(s_1 \cap I_1)}{P(I_1)} = \frac{.12}{.20} = .60$$

$$P(s_2 \mid I_1) = \frac{P(s_2 \cap I_1)}{P(I_1)} = \frac{.05}{.20} = .25$$

FIGURE 21.3 Probability Tree to Compute Posterior Probabilities



$$P(s_3 \mid I_1) = \frac{P(s_3 \cap I_1)}{P(I_1)} = \frac{.03}{.20} = .15$$

Instead of drawing the probability tree to compute the posterior probabilities, we can use Table 21.4, which performs exactly the same calculations. For each state of nature s_j, we multiply the prior probability by the likelihood probability to produce the probability of the intersection. We then add these to determine $P(I_1)$, and we divide the probability of each intersection by $P(I_1)$ to compute the posterior probabilities.

After the probabilities have been revised, we can use them in exactly the same way we used the prior probabilities. That is, we can calculate the expected monetary value of each act:

$$\text{EMV}(a_1) = .60(100{,}000) + .25(100{,}000) + .15(100{,}000) = \$100{,}000$$
$$\text{EMV}(a_2) = .60(-50{,}000) + .25(80{,}000) + .15(180{,}000) = \$17{,}000$$
$$\text{EMV}(a_3) = .60(150{,}000) + .25(90{,}000) + .15(40{,}000) = \$118{,}500$$

Thus, if IMC forecasts s_1, the optimal act is a_3, and the expected monetary value of the decision is $118,500.

TABLE 21.4 Posterior Probabilities for I_1

s_j	$P(s_j)$	$P(I_1 \mid s_j)$	$P(s_j \cap I_1)$	$P(s_j \mid I_1)$
s_1	.2	.60	.12	.12/.20 = .60
s_2	.5	.10	.05	.05/.20 = .25
s_3	.3	.10	.03	.03/.20 = .15
			$P(I_1) = .20$	

As a further illustration, we now repeat the process for I_2 and I_3. If I_2 is the experimental outcome, we have the probabilities shown in Table 21.5. Applying these posterior probabilities to the payoff table, we find the following:

$$\text{EMV}(a_1) = .115(100{,}000) + .770(100{,}000) + .115(100{,}000) = \$100{,}000$$
$$\text{EMV}(a_2) = .115(-50{,}000) + .770(80{,}000) + .115(180{,}000) = \$76{,}550$$
$$\text{EMV}(a_3) = .115(150{,}000) + .770(90{,}000) + .115(40{,}000) = \$91{,}150$$

As you can see, if IMC predicts that s_2 will occur, the optimal act is a_1, with an expected monetary value of $100,000.

If the experimental outcome is I_3, the posterior probabilities are shown in Table 21.6. With this set of posterior probabilities, the expected monetary values are as follows:

$$\text{EMV}(a_1) = .071(100{,}000) + .179(100{,}000) + .750(100{,}000) = \$100{,}000$$
$$\text{EMV}(a_2) = .071(-50{,}000) + .179(80{,}000) + .750(180{,}000) = \$145{,}770$$
$$\text{EMV}(a_3) = .071(150{,}000) + .179(90{,}000) + .750(40{,}000) = \$56{,}760$$

If IMC predicts that s_3 will occur, the optimal act is a_2, with an expected monetary value of $145,770.

At this point, we know the following:

If IMC predicts s_1, then the optimal act is a_3.

If IMC predicts s_2, then the optimal act is a_1.

If IMC predicts s_3, then the optimal act is a_2.

Thus, even before IMC makes its forecast, we know which act is optimal for each of the three possible IMC forecasts. All these calculations can be performed before paying IMC its $5,000 fee. This leads to an extremely important calculation. By per

TABLE 21.5 Posterior Probabilities for I_2

s_j	$P(s_j)$	$P(I_2 \mid s_j)$	$P(s_j \cap I_2)$	$P(s_j \mid I_2)$
s_1	.2	.30	.06	.06/.52 = .115
s_2	.5	.80	.40	.40/.52 = .770
s_3	.3	.20	.06	.06/.52 = .115
			$P(I_2)$ = .52	

TABLE 21.6 Posterior Probabilities for I_3

s_j	$P(s_j)$	$P(I_3 \mid s_j)$	$P(s_j \cap I_3)$	$P(s_j \mid I_3)$
s_1	.2	.10	.02	.02/.28 = .071
s_2	.5	.10	.05	.05/.28 = .179
s_3	.3	.70	.21	.21/.28 = .750
			$P(I_3)$ = .28	

forming the computations just described, the investor can determine whether or not he should hire IMC. That is, he can determine whether or not the value of IMC's forecast exceeds the cost of its information. Such a determination is called a **preposterior analysis.**

Preposterior Analysis

The objective of a preposterior analysis is to determine whether the value of the prediction is greater or less than the cost of the information. *Posterior* refers to the revision of the probabilities, and the *pre* indicates that this calculation is performed before paying the fee.

We begin by finding the expected monetary value of using the additional information. This value is denoted EMV′, which for our example is determined on the basis of the following analysis:

If IMC predicts s_1, then the optimal act is a_3, and the expected payoff is $118,500.

If IMC predicts s_2, then the optimal act is a_1, and the expected payoff is $100,000.

If IMC predicts s_3, then the optimal act is a_2, and the expected payoff is $145,770.

A useful by-product of calculating the posterior probabilities is the set of probabilities of I_1, I_2, and I_3:

$$P(I_1) = .20,\ P(I_2) = .52,\ P(I_3) = .28$$

(Notice that these probabilities sum to 1.) Now imagine that the investor seeks the advice of IMC an infinite number of times. (This is the basis for the expected value decision.) The set of probabilities of I_1, I_2, and I_3 indicates the following outcome distribution: 20% of the time, IMC will predict s_1 and the expected monetary value will be $118,500; 52% of the time, IMC will predict s_2 and the expected monetary value will be $100,000; and 28% of the time, IMC will predict s_3 and the expected monetary value will be $145,770.

The expected monetary value with additional information is the weighted average of the expected monetary values, where the weights are $P(I_1)$, $P(I_2)$, and $P(I_3)$. Hence,

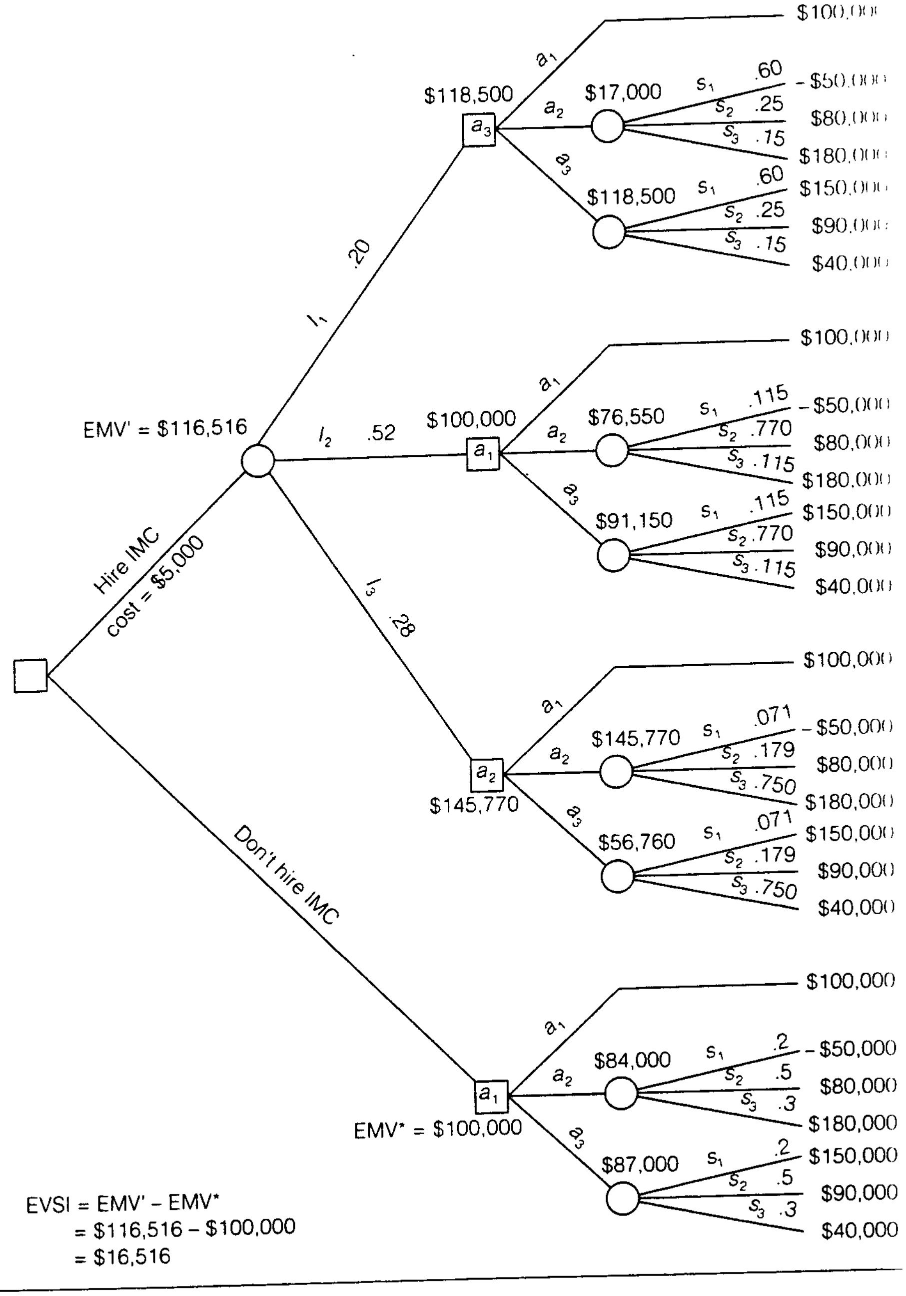
$$\text{EMV}' = .20(118{,}500) + .52(100{,}000) + .28(145{,}770) = \$116{,}516$$

The value of IMC's forecast is the difference between the expected monetary value with additional information (EMV′) and the expected monetary value without additional information (EMV*). This difference is called the **expected value of sample information** and is denoted EVSI. Thus,

$$\text{EVSI} = \text{EMV}' - \text{EMV}^* = \$116{,}516 - \$100{,}000 = \$16{,}516$$

By using IMC's forecast, the investor can make an average additional profit of $16,516, in the long run. Because the cost of the forecast is only $5,000, the investor is advised to hire IMC.

FIGURE 21.4 Complete Decision Tree of Example 21.1



If you review this problem, you'll see that the investor had to make two decisions. The first (chronologically) was whether or not to hire IMC, and the second was which type of investment to make. A decision tree is quite helpful in describing the acts and states of nature in this question. Figure 21.4 provides the complete tree diagram.

EXAMPLE 21.2

A factory produces a small but important component used in computers. The factory manufactures the component in 1,000-unit lots. Because of the relatively advanced technology, the manufacturing process results in a large proportion of defective units. In fact, the quality-control engineer has observed that the percentage of defective units per lot has been either 15% or 35%. In the past year, 60% of the lots have had 15% defectives and 40% have had 35% defectives. The present policy of the company is to send the lot to the customer, replace all defectives, and pay any additional costs. The total cost of replacing a defective unit that has been sent to the customer is $10/unit. Because of the high costs, the company management is considering inspecting all units and replacing the defective units before shipment. The sampling cost is $2/unit, and the replacement cost is $0.50/unit. Each unit sells for $5.

a. Based on the history of the past year, should the company adopt the 100% inspection plan?

b. Is it worthwhile to take a sample of size 2 from the lot before deciding whether to inspect 100%?

Solution

a. The two alternatives are

a_1: No inspection (the current policy)

a_2: 100% inspection

The two states of nature are

s_1: The lot contains 15% defectives.

s_2: The lot contains 35% defectives.

Based on the past year's historical record,

$P(s_1) = .60 \qquad P(s_2) = .40$

The payoff table is constructed as shown in Table 21.7.

TABLE 21.7 Payoff Table for Example 21.2

	a_1	a_2
s_1	$5(1{,}000) - .15(1{,}000)(10) = 3{,}500$	$5(1{,}000) - [(1{,}000)(2) + .15(1{,}000)(.50)] = 2{,}925$
s_2	$5(1{,}000) - .35(1{,}000)(10) = 1{,}500$	$5(1{,}000) - [(1{,}000)(2) + .35(1{,}000)(.50)] = 2{,}825$

The expected monetary values are

$$\text{EMV}(a_1) = .60(3{,}500) + .40(1{,}500) = 2{,}700$$
$$\text{EMV}(a_2) = .60(2{,}925) + .40(2{,}825) = 2{,}885$$

The optimal act is a_2, with EMV* = 2,885.

b. The cost of the proposed sampling is \$4. (The cost of inspecting a single unit is \$2.) In order to determine whether we should sample, we need to calculate the expected value of sample information. That is, we need to perform the preposterior analysis.

The first step of the preposterior analysis is to calculate the likelihood probabilities. There are three possible sample outcomes:

I_0: No defectives in the sample

I_1: One defective in the sample

I_2: Two defectives in the sample

Because the sampling process is a binomial experiment, the likelihood probabilities are calculated by using the binomial probability distribution. (See Table 21.8.)

If I_0 is the sample outcome, the posterior probabilities are calculated as shown in Table 21.9. The expected monetary values in this case are

$$\text{EMV}(a_1) = .720(3{,}500) + .280(1{,}500) = \$2{,}940$$
$$\text{EMV}(a_2) = .720(2{,}925) + .280(2{,}825) = \$2{,}897$$

Therefore, the optimal act is a_1.

If I_1 is the sample outcome, the posterior probabilities are calculated as shown in Table 21.10. The expected monetary values in this case are

$$\text{EMV}(a_1) = .457(3{,}500) + .543(1{,}500) = \$2{,}414$$
$$\text{EMV}(a_2) = .457(2{,}925) + .543(2{,}825) = \$2{,}871$$

TABLE 21.8 Likelihood Probability Table

	$P(I_0 \mid s_j)$	$P(I_1 \mid s_j)$	$P(I_2 \mid s_j)$
s_1 (p = .15)	$P(I_0 \mid s_1) = (.85)^2$ = .7225	$P(I_1 \mid s_1) = 2(.15)(.85)$ = .2550	$P(I_2 \mid s_1)$ = [illegible] = [illegible]
s_2 (p = .35)	$P(I_0 \mid s_2) = (.65)^2$ = .4225	$P(I_1 \mid s_2) = 2(.35)(.65)$ = .4550	$P(I_2 \mid s_2)$ = [illegible] = [illegible]

TABLE 21.9 Posterior Probabilities for I_0

s_j	$P(s_j)$	$P(I_0 \mid s_j)$	$P(s_j \cap I_0)$	$P(s_j \mid I_0)$
s_1	.60	.7225	.4335	.720
s_2	.40	.4225	.1690	.280
			$P(I_0)$ = .6025	

Therefore, the optimal act is a_2.

If I_2 is the sample outcome, the posterior probabilities are calculated as shown in Table 21.11. The expected monetary values in this case are

$$\text{EMV}(a_1) = .216(3{,}500) + .784(1{,}500) = \$1{,}932$$
$$\text{EMV}(a_2) = .216(2{,}925) + .784(2{,}825) = \$2{,}847$$

Therefore, the optimal act is a_2.

We can now summarize these results, as shown in Table 21.12. The expected monetary value with additional information is

$$\text{EMV}' = .6025(2{,}940) + .3350(2{,}871) + .0625(2{,}847) = \$2{,}911$$

The expected value of sample information is

$$\begin{aligned} \text{EVSI} &= \text{EMV}' - \text{EMV}^* \\ &= 2{,}911 - 2{,}885 \\ &= \$26 \end{aligned}$$

Because the expected value of sample information is \$26 and the sampling cost is \$4, the company should take a sample of two units before deciding whether or not to

TABLE 21.10 Posterior Probabilities for I_1

s_j	$P(s_j)$	$P(I_1 \mid s_j)$	$P(s_j \cap I_1)$	$P(s_j \mid I_1)$
s_1	.60	.2550	.153	.457
s_2	.40	.4550	.182	.543
			$P(I_1) = .3350$	

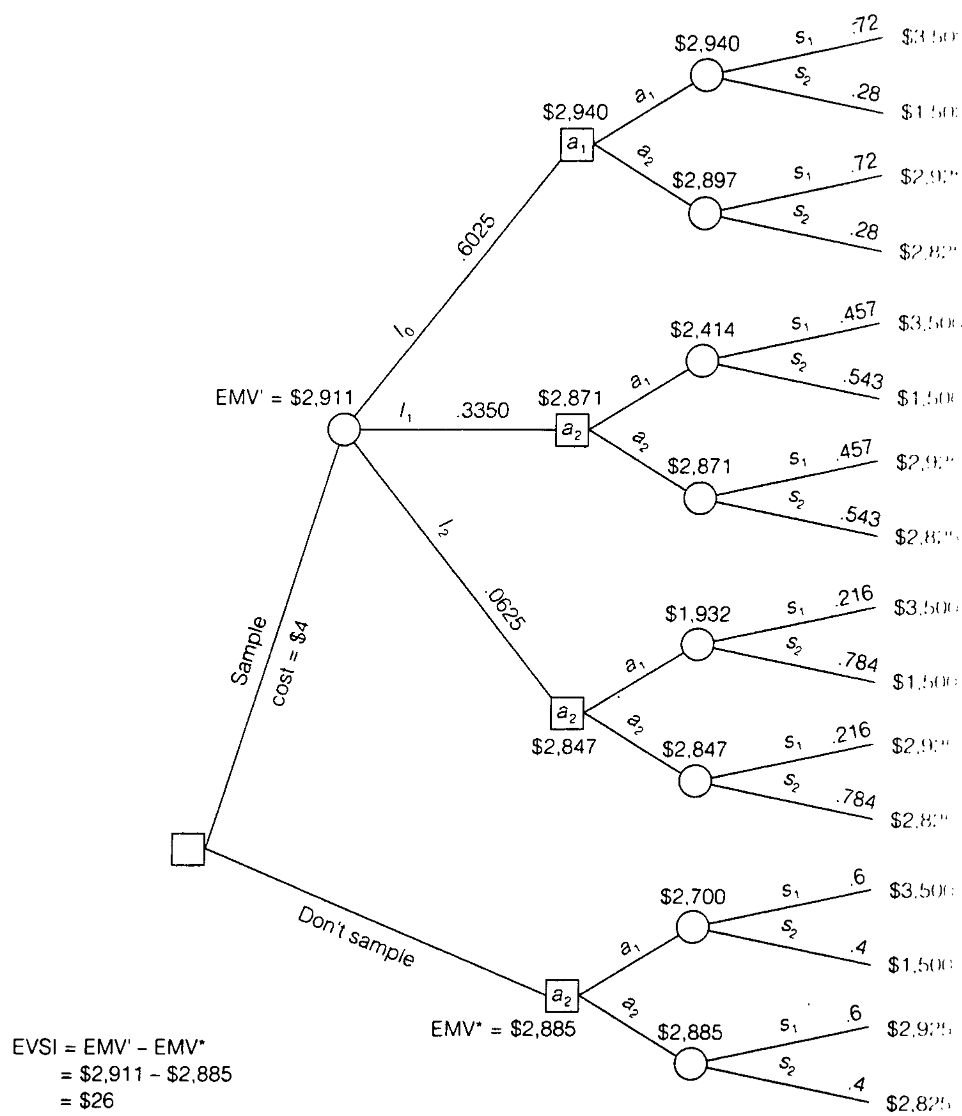
TABLE 21.11 Posterior Probabilities for I_2

s_j	$P(s_j)$	$P(I_2 \mid s_j)$	$P(s_j \cap I_2)$	$P(s_j \mid I_2)$
s_1	.60	.0225	.0135	.216
s_2	.40	.1225	.0490	.784
			$P(I_2) = .0625$	

TABLE 21.12 Summary of Optimal Acts

SAMPLE OUTCOME	PROBABILITY	OPTIMAL ACT	EXPECTED MONETARY VALUE
I_0	.6025	a_1	\$2,940
I_1	.3350	a_2	2,871
I_2	.0625	a_2	2,847

FIGURE 21.5 Decision Tree for Example 21.2



inspect 100%. Figure 21.5 depicts the decision tree for this example. As you can see, the optimal sequence of decisions is as follows:

1. Take a sample of two units.

2. If neither sample unit is defective, continue the current policy of no inspection. If either one or two of the sample units are defective, perform a complete inspection of the lot.

EXERCISES

Learning the Techniques

21.35 Determine the posterior probabilities, given the following prior and likelihood probabilities.

Prior Probabilities:
$P(s_1) = .25, P(s_2) = .40, P(s_3) = .35$

Likelihood Probabilities

	I_1	I_2	I_3	I_4
s_1	.40	.30	.20	.10
s_2	.25	.25	.25	.25
s_3	0	.30	.40	.30

21.36 Calculate the posterior probabilities from the prior and likelihood probabilities that follow.

Prior Probabilities: $P(s_1) = .5, P(s_2) = .5$

Likelihood Probabilities

	I_1	I_2
s_1	.98	.02
s_2	.05	.95

21.37 With the accompanying payoff table and the prior and posterior probabilities computed in Exercise 21.36, calculate the following.

a. the optimal act for each experimental outcome
b. the expected value of sample information

Payoff Table

	a_1	a_2	a_3
s_1	10	18	23
s_2	22	19	15

21.38 Given the accompanying payoff table, prior probabilities, and likelihood probabilities, determine the optimal act for each experimental outcome.

Payoff Table

	a_1	a_2	a_3
s_1	15	17	25
s_2	20	17	10

Prior Probabilities: $P(s_1) = .75, P(s_2) = .25$

Likelihood Probabilities

	I_1	I_2	I_3
s_1	.6	.3	.1
s_2	.1	.2	.7

21.39 Determine the expected value of sample information for Exercise 21.38.

21.40 Draw the decision tree for Exercise 21.38.

21.41 Given the following payoff table, prior probabilities, and likelihood probabilities, find the expected value of sample information.

Payoff Table

	a_1	a_2
s_1	60	90
s_2	90	90
s_3	150	90

Prior Probabilities:
$P(s_1) = 1/3, P(s_2) = 1/3, P(s_3) = 1/3$

Likelihood Probabilities

	I_1	I_2
s_1	.7	.3
s_2	.5	.5
s_3	.2	.8

21.42 Repeat Exercise 21.41, with the following prior probabilities:

$P(s_1) = .5, P(s_2) = .4, P(s_3) = .1$

21.43 Repeat Exercise 21.41, with the following prior probabilities:

$P(s_1) = .90, P(s_2) = .05, P(s_3) = .05$

21.44 What conclusions can you draw about the effect of the prior probabilities on EVSI from Exercises 21.41 through 21.43?

21.45 For the following payoff table, prior probabilities, and likelihood probabilities, determine the expected value of sample information.

Payoff Table

	a_1	a_2
s_1	18	20
s_2	24	20

Prior Probabilities: $P(s_1) = .7$, $P(s_2) = .3$

Likelihood Probabilities

	l_1	l_2
s_1	.8	.2
s_2	.3	.7

21.46 Draw the decision tree for Exercise 21.45.

21.47 Repeat Exercise 21.45, with the following likelihood probabilities.

	l_1	l_2
s_1	.96	.04
s_2	.02	.98

21.48 Repeat Exercise 21.45, with the following likelihood probabilities.

	l_1	l_2
s_1	.6	.4
s_2	.6	.4

21.49 What conclusions can you draw about the effect of the likelihood probabilities on EVSI from Exercises 21.45, 21.47, and 21.48?

Self-Correcting Exercise

(See Appendix 21.A for the solution.)

21.50 Mr. Jones is attempting to determine where he should invest his $100,000. There are three possible investments: a_1, a_2, and a_3. The events of interest are the various states of the economy. Table E21.50 shows the alternatives, the states of nature, and the payoffs.

The investor assigns the following probabilities.

$$P(s_1) = .2, P(s_2) = .5, P(s_3) = .3$$

Ace Economic Forecasters (AEF) will make a prediction on the state of the economy for $600. As a measure of its past reliability, AEF offers the accompanying set of likelihood probabilities.

	l_1 (predict recession)	l_2 (predict stable economy)	l_3 (predict upturn)
s_1	.5	.5	0
s_2	.2	.7	.1
s_3	.1	.3	.6

Perform a preposterior analysis to determine whether AEF should be hired.

21.51 In order to improve his decision-making capability, the electronics products manufacturer in Exercise 21.23 performs a survey of potential buyers of compact disk players. He describes the product to 25 individuals, 3 of whom say they would buy it. Using this additional information (together with the prior probabilities), determine whether the new product should be produced.

21.52 There is a garbage crisis in North America—too much garbage and no place to put it. As a consequence, the idea of recycling has become quite popular. A waste-management company in a large city is willing to begin recycling newspapers, aluminum cans, and plastic containers. However, it is profitable to do so only if a sufficiently large proportion of households is willing to participate. In this city, 1 million households are potential recyclers. After some analysis, it was determined that, for every 1,000 households that participate in the program, the contribution to profit is $500. It was also discovered that fixed costs are $55,000 per year. It is believed that 50,000, 100,000, 200,000, or 300,000 households will participate, with probabilities .5, .3, .1, and .1, respectively. A preliminary survey was per-

TABLE **E21.50**

	a_1	a_2	a_3
s_1 (recession)	$1,000	−$ 2,000	−$10,000
s_2 (stable economy)	4,000	7,000	8,000
s_3 (economic upturn)	8,000	12,000	15,000

TABLE **E21.54** Likelihood Probabilities

	I_1 (predict low demand for electricity)	I_2 (predict moderate demand for electricity)	I_3 (predict high demand for electricity)
s_1	.5	.3	.2
s_2	.3	.6	.1
s_3	.2	.2	.6

formed wherein 25 households were asked if they would be willing to be part of this recycling program. Suppose only 3 of the 25 respond affirmatively; incorporate this information into the decision-making process to decide whether the waste-management company should proceed with the recycling venture.

21.53 Repeat Exercise 21.52, given that 12 out of 100 households respond affirmatively.

21.54 Suppose that in Exercise 21.20 a consultant offers to analyze the problem and predict the amount of electricity required by the new community. In order to induce the electric company to hire her, the consultant provides the set of likelihood probabilities given in Table E21.54. Perform a preposterior analysis to determine the expected value of the consultant's sample information.

21.55 In Exercise 21.33, suppose that it is possible to survey radio listeners to determine whether they would tune in to the station if the format changed to rock and roll. What would a survey of size 2 be worth?

21.56 Suppose that in Exercise 21.55 a random sample of 25 radio listeners revealed that 2 people would be regular listeners of the station. What is the optimal decision now?

SECTION 21.6 Summary

The objective of decision analysis is to select the optimal act from a list of alternative acts. We define as optimal the act with the largest expected monetary value or smallest expected opportunity loss. The expected values are calculated after assigning prior probabilities to the states of nature. The acts, states of nature, and their consequences may be presented in a payoff table, an opportunity loss table, or a decision tree.

We also discussed a method by which additional information in the form of an experiment can be incorporated in the analysis. This method involves combining prior and likelihood probabilities to produce posterior probabilities. The preposterior analysis allows us to decide whether or not to pay for and acquire the experimental outcome. That decision is based on the expected value of sample information and on the sampling cost.

Important Terms

Alternative acts	Expected monetary value
States of nature	Expected opportunity loss
Payoff table	Likelihood probabilities
Opportunity loss table	Posterior probabilities
Decision tree	Preposterior probabilities
Prior probabilities	Expected value of sample information

SUPPLEMENTARY EXERCISES

21.57 The MacTell toy company is considering producing a new toy—the R2-D2 robot. The company is considering four prototype designs for the robot. The prototype represents a different technology for the moving parts, all of which are powered by a small electric motor using batteries.

The MacTell company is uncertain about the demand for the R2-D2 robot, but it feels that one of the following states of nature will occur:

Light demand (25,000 units)
Moderate demand (100,000 units)
Heavy demand (150,000 units)

Using revenues, variable costs, and fixed costs, the MacTell company calculated the payoffs shown in Table E21.57. After some consideration, the company president assigned the following probabilities to the states of nature:

P(light demand) = .2
P(moderate demand) = .6
P(heavy demand) = .2

Based on the expected monetary value, which design should be selected?

21.58 A pottery maker hand-makes beautiful vases. The materials for each vase cost her \$3. By market-testing the vases at different prices, she obtained the frequency estimates of daily demand given in Table E21.58. Which price should she charge, based on the expected monetary value decision? (HINT: Draw a decision tree.)

21.59 The president of an automobile battery company must decide which one of three new types of batteries to produce. The fixed and variable costs of each battery are shown in the accompanying table.

BATTERY	FIXED COST	VARIABLE COST (per unit)
1	\$ 900,000	\$20
2	1,150,000	17
3	1,400,000	15

The president believes that demand will be 50,000, 100,000, or 150,000 batteries, with probabilities .3, .3, and .4, respectively. The selling price of the battery will be \$40.

a. Determine the payoff table.
b. Determine the opportunity loss table.
c. Find the expected monetary value for each act, and select the optimal one.
d. What is the most the president should be willing to pay for additional information about demand?

21.60 Credibility is often the most effective feature of an advertising campaign. Suppose that, for a particular advertisement, 32% of people surveyed currently believe what the ad claims. A marketing manager believes that for each 1-point

TABLE E21.57 Potential Payoffs, by Design (\$1,000s)

STATES OF NATURE	DESIGN 1	DESIGN 2	DESIGN 3	DESIGN 4
Light demand	30	0	−100	−300
Moderate demand	400	450	400	300
Heavy demand	600	750	800	700

TABLE E21.58

DEMAND (at \$6)	PROBABILITY	DEMAND (at \$7)	PROBABILITY
7	.15	5	.10
8	.25	6	.20
9	.40	7	.30
10	.20	8	.40

increase in that percentage, annual sales will increase by $1 million. For each 1-point decrease, annual sales will decrease by $1 million. The manager believes that a change in the advertising approach can influence the ad's credibility. The probability distribution of the potential changes is listed next.

PERCENTAGE CHANGE	PROBABILITY
−2	.1
−1	.1
0	.2
+1	.3
+2	.3

If for each dollar of sales the profit contribution is 10¢ and the overall cost of changing the ad is $58,000, should the ad be changed?

21.61 Suppose that in Exercise 21.60 it is possible to perform a survey to determine the percentage of people who believe the ad. What would a sample of size 1 be worth?

21.62 Suppose that in Exercise 21.61 a sample of size 5 showed that only one person believes the new ad. In light of this additional information, what should the manager do?

21.63 After the first 78 games of an 80-game hockey season, Wayne Grootsky has scored 98 goals. As an additional incentive, the owner of his team (the L.A. Filter Kings) has offered him his choice of one of the following bonus plans:

1. $100,000 for each additional goal up to a maximum of 2
2. $50,000 for each additional goal
3. nothing for his 99th goal, but $300,000 if he scores at least 2 more goals

Lately he has suffered a scoring slump, averaging only .9 goal per game. However, his goal scoring remains random, and the goals he scores are independent of one another.

Identify the acts from which Wayne must choose and the events that would affect the outcome, and construct the payoff table. (Treat probabilities of less than .01 as being equal to zero.)

21.64 Max the bookie is trying to decide how many telephones to install in his new bookmaking operation. Because of heavy police activity, he cannot increase or decrease the number of telephones once he sets up his operation. He has narrowed the possible choices to three: he can install 25, 50, or 100 telephones. His profit for one year (the usual length of time he can remain in business before the police close him down) depends on the average number of calls he receives. (Calls occur randomly and independently of one another.) After some deliberation, he concludes that the average number of calls per minute can be .5, 1.0, or 1.5, with probabilities of .50, .25, and .25, respectively. Max then produces the payoffs given in Table E21.64.

Max's assistant, Lefty (who actually attended a business school for two years), points out that Max may be able to get more information by observing a competitor's similar operation. However, he will be able to watch for only 10 minutes, and doing so will cost him $4,000. Max determines that if he counts fewer than 8 calls, that would be a low number; at least 8 but fewer than 17 would be a medium number; and at least 17 would be a large number of calls. Max also decides that, if the experiment is run, he will record only whether there is a small, medium, or large number of calls. Help Max by performing a preposterior analysis to determine whether the sample should be taken. Conclude by specifying clearly what the optimal strategy is. (HINT: The number of telephone calls is Poisson-distributed.)

21.65 The Megabuck Computer Company is thinking of introducing two new products. The first, Model 101, is a small computer designed specifically for children between the ages of 8 and 16. The second, Model 202, is a medium-size computer suitable for managers. Because of limited production capacity, Megabuck has decided to produce only one of the products.

TABLE **E21.64** Payoff Table ($1,000s)

	a_1 (25 telephones)	a_2 (50 telephones)	a_3 (100 telephones)
s_1 (μ = .5)	50	30	20
s_2 (μ = 1.0)	50	60	40
s_3 (μ = 1.5)	50	60	80

The profitability of each model depends on the proportion of the potential market that would actually buy the computer. For Model 101, the size of the market is estimated at 10 million, while for Model 202, the estimate is 3 million.

After careful analysis, the management of Megabuck has concluded that the percentage of buyers of Model 101 is 5%, 10%, or 15%. The respective profits are given in the accompanying table.

PERCENTAGE WHO BUY MODEL 101	NET PROFITS ($millions)
5%	20
10	100
15	210

An expert in probability from the local university estimated the probability of the percentages as Prob(5%) = .2, Prob(10%) = .4, and Prob(15%) = .4.

A similar analysis for Model 202 produced the following table.

PERCENTAGE WHO BUY MODEL 202	NET PROFITS ($millions)
30%	70
40	100
50	150

For this model, the expert estimated the probabilities as Prob(30%) = .1, Prob(40%) = .4, and Prob(50%) = .5

a. Based on this information, and with the objective of maximizing expected profit, which model should Megabuck produce?

b. In order to make a better decision, Megabuck sampled 10 potential buyers of Model 101 and 20 potential buyers of Model 202. Only 1 of the 10 wished to purchase the Model 101, while 9 of the 20 indicated that they would buy Model 202. Given this information, revise the prior probabilities and determine which model should be produced.

21.66 A major movie studio has just completed its latest epic, a musical comedy about the life of Attila the Hun. Because the movie is different (no sex or violence), the studio is uncertain about how to distribute it. The studio executives must decide whether to release the movie to North American audiences or to sell it to a European distributor and realize a profit of $12 million. If the movie is shown in North America, the studio profit depends on its level of success, which can be classified as excellent, good, or fair. The payoffs and the prior subjective probabilities of the success levels are shown in the accompanying table.

SUCCESS LEVEL	PAYOFF	PROBABILITY
Excellent	$33 million	.5
Good	12 million	.3
Fair	−15 million	.2

Another possibility is to have the movie shown to a random sample of North Americans and use their collective judgment to help the studio make a decision. These judgments are categorized as "rave review," "lukewarm response," or "poor response." The cost of the sample is $100,000.

The sampling process has been used several times in the past. The likelihood probabilities relating the audience judgments and the movie's success level are shown in Table E21.66. Perform a preposterior analysis to determine what the studio executives should do.

21.67 Laurier Industries is considering whether or not to launch a new product. Its experience indicates that any new product has a 5% chance of being a "great success," a 20% chance of being a "moderate success," a 30% chance of being a "poor success," and a 45% chance of being an "outright failure."

In keeping with the firm's usual practice, a market study was carried out for the new product. This market study indicated that the product would be a moderate success. Based on past results, management would expect 30% of all

TABLE E21.66 Likelihood Probabilities

SUCCESS LEVEL	JUDGMENTS: Rave Review	Lukewarm Response	Poor Response
Excellent	.8	.1	.1
Good	.5	.3	.2
Fair	.4	.3	.3

"great success" products, 45% of all "moderate success" products, 20% of all "poor success" products, and 10% of all "outright failure" products to yield a market study result indicating "moderate success."

Management's criterion for launching the product is that the probability that the product will be a moderate or great success must be greater than .45. If the probability of failure is greater than .5, the new product will be dropped. Otherwise, another market study will be initiated.

What should be done with the new product? Justify your answer.

21.68 A manufacturer of microcomputers is in the process of deciding whether or not to purchase a lot of 1 million monitors at $65 each. The current policy is to manufacture the monitors at a cost of $70. When the microcomputer manufacturer produces his own monitors, however, the defective rate is known to be 6%. The defective rate of the supplier's monitors is unknown, but the distribution of percent defectives shown in the accompanying table has been estimated.

PERCENT DEFECTIVE	PROBABILITY
5%	.1
10	.2
15	.4
20	.3

The cost to the microcomputer manufacturer of replacing a defective monitor is $50. Should the manufacturer purchase the 1 million monitors or produce them himself?

21.69 In Exercise 21.68, suppose that it is possible to take a random sample of monitors from the lot of 1 million to determine how many are defective.

a. What would a sample of size 1 be worth?

b. What would a sample of size 2 be worth?

21.70 In Exercise 21.68, suppose that a sample of 20 monitors was selected from the lot and that 4 were found to be defective. Should the manufacturer purchase the 1 million monitors or produce them himself?

APPENDIX 21.A

Solutions to Self-Correcting Exercises

21.6 Let a_0 = Bake 0 cakes

a_1 = Bake 1 cake

a_2 = Bake 2 cakes

a_3 = Bake 3 cakes

s_0 = Demand for 0 cakes

s_1 = Demand for 1 cake

s_2 = Demand for 2 cakes

s_3 = Demand for 3 cakes

Payoff Table

	ACTS			
STATES OF NATURE	a_0	a_1	a_2	a_3
s_0	0	−3	−6	−9
s_1	0	5	2	−1
s_2	0	5	10	7
s_3	0	5	10	15

21.20

a. Expected cost (build small plant)

= .15(220) + .55(330) + .30(440) = 346.5

Expected cost (build medium plant)

= .15(300) + .55(320) + .30(390) = 338

Expected cost (build large plant) = 350

The act with the largest expected monetary value is the act with the smallest expected cost. Thus, the act with the largest expected monetary value is to build a medium plant.

b. Opportunity Loss Table

	ACTS		
STATES OF NATURE	a_1	a_2	a_3
s_1	0	80	130
s_2	10	0	30
s_3	90	40	0

c. $\text{EOL}(a_1) = .15(0) + .55(10) + .30(90) = 32.5$
$\text{EOL}(a_2) = .15(80) + .55(0) + .30(40) = 24$
$\text{EOL}(a_3) = .15(130) + .55(30) + .30(0) = 36$

The optimal act is a_2 (build medium plant).

21.30 Let a_1 = Buy footballs
a_2 = Don't buy footballs
s_1 = Demand of 10,000
s_2 = Demand of 30,000
s_3 = Demand of 50,000

Payoff Table ($1,000s)

	ACTS	
STATES OF NATURE	a_1	a_2
s_1	−75	0
s_2	25	0
s_3	125	0

$\text{EMV}(a_1) = .2(-75) + .5(25) + .3(125) = 35$
$\text{EMV}(a_2) = 0$
$\text{EMV}^* = 35$
$\text{EPPI} = .2(0) + .5(25) + .3(125) = 50$
$\text{EVPI} = 50 - 35 = 15$

The maximum price the owner should pay is $15,000.

21.50 Based on prior probabilities,

$\text{EMV}(a_1) = .2(1{,}000) + .5(4{,}000) + .3(8{,}000) = 4{,}600$
$\text{EMV}(a_2) = .2(-2{,}000) + .5(7{,}000) + .3(12{,}000) = 6{,}700$
$\text{EMV}(a_3) = .2(-10{,}000) + .5(8{,}000) + .3(15{,}000) = 6{,}500$

Optimal act is a_2. $\text{EMV}^* = 6{,}700$.

If prediction is I_1:

s_j	$P(s_j)$	$P(I_1 \mid s_j)$	$P(I_1 \cap s_j)$	$P(s_j \mid I_1)$
s_1	.2	.5	.10	.435
s_2	.5	.2	.10	.435
s_3	.3	.1	.03	.130
			$P(I_1) = .23$	

$\text{EMV}(a_1) = .435(1{,}000) + .435(4{,}000) + .130(8{,}000) = 3{,}215$
$\text{EMV}(a_2) = .435(-2{,}000) + .435(7{,}000) + .130(12{,}000) = 3{,}735$
$\text{EMV}(a_3) = .435(-10{,}000) + .435(8{,}000) + .130(15{,}000) = 1{,}080$

Optimal act is a_2.

If prediction is I_2:

s_j	$P(s_j)$	$P(I_2 \mid s_j)$	$P(I_2 \cap s_j)$	$P(s_j \mid I$
s_1	.2	.5	.10	.185
s_2	.5	.7	.35	.648
s_3	.3	.3	.09	.167
			$P(I_2) = .54$	

$\text{EMV}(a_1) = .185(1{,}000) + .648(4{,}000) + .167(8{,}000) = 4{,}113$
$\text{EMV}(a_2) = .185(-2{,}000) + .648(7{,}000) + .167(12{,}000) = 6{,}170$
$\text{EMV}(a_3) = .185(-10{,}000) + .648(8{,}000) + .167(15{,}000) = 5{,}839$

Optimal act is a_2.

If prediction is I_3:

s_j	$P(s_j)$	$P(I_3 \mid s_j)$	$P(I_3 \cap s_j)$	$P(s_j \mid$
s_1	.2	0	0	
s_2	.5	.1	.05	.21
s_3	.3	.6	.18	.78
			$P(I_3) = .23$	

$\text{EMV}(a_1) = 0(1{,}000) + .217(4{,}000) + .783(8,$
$= 7{,}132$
$\text{EMV}(a_2) = 0(-2{,}000) + .217(7{,}000) + .783(12{,}000) = 10{,}915$
$\text{EMV}(a_3) = 0(-10{,}000) + .217(8{,}000) + .783(15{,}000) = 13{,}481$

Optimal act is a_3.

$\text{EMV}' = .23(3{,}735) + .54(6{,}170) + .23(13{,}4$
$= 7{,}291.48$
$\text{EVSI} = 7{,}291.48 - 6{,}700 = \$591.48$

Because the cost of additional information is $600, AEF should not be hired.