

$$\text{Diff} = .5(U(15) + U(20))$$

Suppose if $U(20) > U(15)$

$$= .5(U(15) + [U(15) + (U(20) - U(15))])$$

$$= .5(2U(15) + [U(20) - U(15)])$$

$$= U(15) + .5[U(20) - U(15)]$$

$$\tan \angle A = \frac{B}{C} = \frac{U(20) - U(15)}{20 - 15}$$

$$\text{but also } \tan \angle A = \frac{.5B}{.5C} = \frac{D}{.5C}$$

$$\text{so } D = .5B = .5[U(20) - U(15)]$$

$$\text{since } E = U(15), E+D = U(15) + .5[U(20) - U(15)] = .5U(15) + .5U(20) = E(U)_{NI}$$

ii. With insurance

Suppose that an insurance policy is offered so that if ill the person receives payoff of \$5,000. The premium is actuarially fair (i.e. reflects the underlying probabilities) so that over the entire insured population $E(\text{payoff}) = \text{premium}$. But only those getting sick get paid off so $E(\text{payoff}) = p(\text{payoff}) * \text{payoff} = .5 * 5,000 = \$2,500$.

Now calculate $E(Y)$ for everyone in the insurance pool. If ill income is \$15,000 but payoff is \$5,000 but the premium for the insurance must still be paid. So the income of the ill person is $[15,000 + 5,000 - 2,500] = \$17,500$. If not ill income is \$20,000 but premium must be paid so this person's income is $[20,000 - 2,500] = \$17,500$. The effect of the insurance has been to turn an uncertain income into a certain income. This means that $E(U)$ with insurance equals $U(17,500)$.

Note that if the two groups are of equal size and they are combined
 $pA+B(ill) = .5 * pA(ill) + .5 * pB(ill) = .5 * .2 + .5 * .8 = .5$ and
 $pA+B(not\ ill) = .5 * pA(not\ ill) + .5 * pB(not\ ill) = .5 * .8 + .5 * .2 = .5$