$$E\left[\frac{\sum(X_i-\bar{X})^2}{n}\right] = \tfrac{1}{n}E\left[\sum(X_i-\bar{X})^2\right] = \tfrac{1}{n}E\left[\sum(\bar{X}_i-\mu)^2 - n(\bar{X}-\mu)^2\right]$$
$$= \tfrac{1}{n}\left[\sum E[(X-\mu)]^2 - nE\left[(\bar{X}-\mu)^2\right]\right]$$

Since the expected values of a sum of random variables is the sum of the expected values of the variables.

Note that the first term expected value is just the expression for the variance of x and the second term expected value is just the expression for the variance of $\bar{X}$ (recalling that $E(\bar{X}) = \mu$. So

$$E\left[\frac{\sum(X_i-\bar{X})^2}{n}\right] = \tfrac{1}{n}\left(\sum\sigma_x^2 - n\sigma_{\bar{x}}^2\right) = \tfrac{1}{n}\left(n\sigma_x^2 - \frac{n\sigma_{\bar{x}}^2}{n}\right) = \sigma_x^2\frac{n-1}{n}$$

Thus $\frac{\sum(X_i-\bar{X})^2}{n}$ is a biased estimator by the amount $\frac{n-1}{n}$. If however we multiply $\frac{\sum(X_i-\bar{X})^2}{n}$ by $\frac{n}{n-1}$ and take the expected value, we get

$$E\left[\frac{\sum(X_i-\bar{X})^2}{n} * \frac{n}{n-1}\right] = \frac{n}{n-1}E\left[\frac{\sum(X_i-\bar{X})^2}{n}\right] = \left(\frac{n}{n-1}\right)\sigma_x^2\left(\frac{n-1}{n}\right) = \sigma_x^2$$

So $S_x^2 = \frac{\sum(X_i-\bar{X})^2}{n} * \frac{n}{n-1} = \frac{\sum(X_i-\bar{X})^2}{n-1}$

is an unbiased estimator of the population variance.

$E(S_x^2) = \sigma^2$