EXTERNAL APPENDICES TO CHAPTER 2

Appendix X-2.1: OTHER POPULATION FORECASTS

The economic and demographic literatures feature a variety of population forecasts for 2050. The most widely cited are those of the Census Bureau and the Social Security Administration. Unlike the Lee-Tuljapurkar projections that are used in Chapter 2, these estimates are not very explicit about how the high and low estimates are calculated.

Table X-2.1: Governmental Forecasts of Future Population Growth and Elderly-Dependency Ratios

	Average annual population growth. 2000 to 2050	Elderly-Dependency Ratios 2010	2030	2050
Census Bureau				
Low	0.13%	21.8%	36.3%	35.2%
Medium	0.74	22.3	37.6	37.9
High	1.24	22.7	38.3	39.9
Social Security Administration				
High Social Security cost	0.42	21.8	38.3	44.6
Medium Social Security cost	0.59	21.4	35.4	37.0
Low Social Security cost	0.80	20.8	32.5	30.9

Note: The elderly-dependency-ratio is the ratio of the population over 64 to those in the working ages from 20 through 64.

Census Bureau estimates come from U.S. Department of Commerce, Census Bureau (1996). For their low and high forecasts, I had to make several minor adjustments to achieve comparability with the other estimates in the text; nevertheless, these adjustments do not affect the unexpected finding that the elderly dependency ratios have a direct, rather than inverse, relation to the population growth rate. The SSA estimates come from U.S. Social Security Administration (1999, Table II-H-1).

Appendix X-2.2: SOME MATHEMATICAL PROPERTIES OF THE SIMULATION MODEL: FOUR NOTES

Note A

The discussion in the text is based on intuitive arguments and numbers derived from the simulations. Nevertheless, if we assume a world without an interest rate, the various results can be derived from a simple algebraic model which provides more rigor to the argument that net saving will fall between 2000 and 2050.

Let S_t = net saving. The saving of active workers = $(F_t\ Y_t)\ (a_t\ A_t)$, where F = the saving rate, Y = average income, a_t = percentage of adult population who are active workers, and A = adult population. The expression in the first set of parentheses is the saving of one worker and the expression in the second set of parentheses the number of active workers. Dissaving by retired workers = consumption by retired workers = $[D\ (1 - F_t)\ Y_t\ z_t]\ [(1-a_t)\ A_t]$, where D = consumption replacement ratio, and z = ratio of income on which saving decisions of retired workers were based to current income of active workers. The expression in the first set of brackets is the dissaving of a single worker, and the expression in the second set of brackets is the number of retired workers. Thus in any given year,

$S_t = (F_t\ Y_t)\ (a_t\ A_t) - [D\ (1 - F_t)\ Y_t\ z_t]\ [\ (1 - a_t)\ A_t\ ]$. Since we are interested in the net saving ratio, (to total income), this expression can be arranged for easier analysis:

(1) $(S_t /\ a_t\ A_t\ Y_t) = F_t - [D\ (1 - F_t)\ z_t]\ [(1-a_t)/a_t]$.

Given the assumed annual growth in income of 1.8% and assuming that the population in each year cohort between retirement and death is the same, $z_t = .60$. Since D = 1, the expression reduces to:

(1a) $(S_t /\ a_t\ A_t\ Y_t) = F_t - [.6\ (1 - F_t)]\ [(1-a_t)/a_t]$.

Since F_t depends on the age of retirement and the life expectancy, two variables which can change over time, it is necessary to know how F will change when these two variables change. Since lifetime saving (total average annual savings times number of work years) = total dissaving in retirement (annual consumption times number of retirement years), the situation can be easily modeled. Let k = percentage of adult years spent working and K = total adult years. Then F Y k K = [D (1 - F_t) Y] [(1-k) K]. Rearranging and simplifying:

(2) $F = D\,(1\text{-}k)\,[(1 - D) / k]$.

Given the assumptions of the model, the expression in brackets is close to unity and can be simplified to:

(2a) $F = (1\text{-}k)$.

Since life expectancy rises from 80 to 85 and the retirement age from 65 to 70, k changes from 0.727 to 0.750, and F falls from 0.273 to 0.250 in the simulations. This decline should be obvious. since the number of retirement years is the same, but workers have more years to accumulate the necessary savings.

Given the assumptions of the model, the question is how the aggregate changes in the saving rate can be derived. From the L-T data we determine that variable “a” falls from 0.820 to 0.766 and, as a result, (1-a)/a rises from 0.219 to 0.305 (in the simulations it falls somewhat less because the assumptions in the simulation are somewhat different). Because the expressions in both of the brackets in equation 1a are increasing and because F_t is falling, the saving rate falls.

Although the numerical results are sensitive to the population estimates, they do not affect the qualitative results. With the SSA estimates, the same fall in (1-a)/a occurs, but with the Census estimates there is a slight increase. Nevertheless, in the latter case the result of multiplying the two bracketed expressions in equation 1a still shows an increase.

Note B

In equation 1 in Note A, the only expression that is affected by a change in the population growth rate is [(1-a)/a], where a = the proportion of adults who are active workers. With the falling elderly-dependency ratio, this bracketed expression rises along with the saving rate, other factors remaining constant.

Note C

In equation 1 in Note A, z (the ratio of the income on which a retired worker bases dissaving to the income of a current worker) declines and this, in turn, raises the aggregate saving rate.

Note D

This can be shown by differentiating equation 2 in Note A:

(* F /* D) = [(1-k)/(k - D k + D)] - D (1-k) [1/(k - D k + D)]^2 (1-k). After rearrangement and simplification this yields: * F /* D) = [(1-k)k]/[D (1-k) +k]^2. Both numerator and denominator are positive, and since k (the percentage of an adult life span spent working) lies between 0 and 1, the numerator is positive and less than unity/ So 0 < * F /* D < 1, and saving rises more slowly than the consumption replacement ratio.

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